BACHELOR OF SCIENCE EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

MATHEMATICS (SUBSIDIARY)

Calculus - II

Paper : 4S

Time : Two hours   Full Marks : 50

PART - I (30 marks)

Answer any *three* questions.

1. (a) If $u = x f\left(\dfrac{y}{x}\right) + g\left(\dfrac{y}{x}\right)$ then find the value of

   $$x^2\, u_{xx} + 2xy\, u_{xy} + y^2 u_{jy}$$ 5

   (b) Examine the differentiability for the function

   $$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases} \quad \text{at } (0,0)$$ 5

2. (a) Evaluate $\iint\limits_{D} x^2 y\ dxdy$ over the first quadrant of the

   ellipse $\dfrac{x^2}{\alpha^2} + \dfrac{y^2}{\beta^2} = 1$.   5

(Turn Over)

(2)

(b) Examine the continuity for the function

$$f(x,y)=\begin{cases} \dfrac{x^3+y^3}{x-y} & if\ x\neq y \\ 0 & if\ x=y \end{cases}\quad at\ (0,0)$$

5

3. (a) If u, v, w are functions of x, y, z given by $x = u + v + w$, $y = u^2 + v^2 + w^2$, $z = u^3 + v^3 + w^3$, then show that

$$\frac{\partial^2 u}{\partial x} = \frac{vw(w-v)}{(u-v)(v-w)(w-u)}$$

5

(b) Find the moment of inertia of a uniform triangular lamina about its one side.   5

4. (a) Evaluate $\iint\limits_{D} e^{\frac{y-x}{y+x}}\,dxdy$, where D is the region bounded by the triangle with vertices (0,0), (1,0), (0,1).   4

(b) If $u = x^m y^n z^p$, where $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 1$

$a,b,c,m,n,p, \in R^+$, find maximum or minimum value of u.   6

(3)

PART - II (20 marks)

Answer any *two* questions.   10x2=20

5. Solve :

(a) $y^2\,dx + \left(x - \dfrac{1}{y}\right)dy = 0$ 5

(b) $y(2xy + e^x)\,dx = e^x\,dy$ 5

6. Solve :

(a) $(xy^3 + y)\,dx + 2\,(x^2y^2 + x + y^4)dy = 0.$ 5

(b) $(D^2 - 7D + 6)y = (x-2)e^x$ 5

7. (a) Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter and c is a constant.   5

(b) Solve : $xy\,dx + (2x^2 + 3y^2 - 20)dy = 0.$ 5

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