10. Show that the Redal equation of the ellipse $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=1$ w.r.t a focus is $\frac{b^2}{p^2}=\frac{2a}{r}-1$

11. Find the radius of curvature at any point P of the catenary $y=c\cosh\left(\frac{x}{c}\right)$ and show that PC=PG, where C is the centre of curvature at P and G is the point of intersection of the normal at P with the x-axis.

First B.Sc. Examination, 2019

(1st year1st Semester)

Calculus

Paper - 1.1

Time : Two hours　　　　Full Marks : 50

Use a separate answerscript for each Part.

Part -I (30 Marks)

Answer any three questions

(Symbols/Notation/ have their usual meaning)

1. a) Does $\lim_{x\to 0}\left(\operatorname{Sin}\frac{1}{x}+x\operatorname{Sin}\frac{1}{x}\right)$ exist?

   b) Let the function f be defined by

$$f(x) = \begin{cases} \frac{1-\cos x}{x^2}, \text{for } x \neq 0 \\ 1, \quad \text{for } x = 0 \end{cases}$$

   Is f contimuous at x=0 ? Explain.

   c) Find the nature of dis continuity of $f(x)=\frac{1}{x}\sin\frac{1}{x}$ at x=0

4+4+2

2. a) Use mean value theorem to prove that $0<\frac{1}{x}\log\frac{e^x-1}{x}<1$, for $x > 0$

b) Evaluate the limit $\lim_{n\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$ 5+5

3. a) State Euler's theorem for homageneous function of two ariables.

If $v = \text{Sin}^{-1}\sqrt{\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}}$, the prove that

$$x^2\frac{\partial^2 v}{\partial x^2} + 2xy\frac{\partial^2 v}{\partial x \partial y} + y^2\frac{\partial^2 v}{\partial y^2} = \frac{\tan v}{12}\left(\frac{13}{12} + \frac{\tan^2 v}{12}\right)$$

b) If $x^x y^y z^z = K$ then show that at x=y=z

$$\frac{\partial^2 z}{\partial x \partial y} = -\left(x \log_e^x\right)^{-1}$$ 5+4

4. a) Given that $f(x,y) = xy\frac{x^2 - y^2}{x^2 + y^2}, (x,y) \neq (0,0)$

$= 0, \quad (x,y) = (0,0).$

Then show that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

b) Examine the continuity of the function

$$f(x,y) = \begin{cases} (px+qy)\sin\frac{x}{y}, & y \neq 0 \\ 0, & y=0 \end{cases}$$ 5+5

5. a) State and Prove fundamental therum of inteqral calculus.

b) Show that function g difined by

$$g(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x=0 \end{cases}$$

is differentiable, everywhere, but the derived function is not continuous at x=0 5+5

Part-II (20 marks)

Answer *any four* questions

5×4=20

6. For any curve $r = f(\theta)$, prove that $\frac{ds}{dv} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$.

7. Define asymptote of a curve. Examine the asymptotes, if any, of the curve $y - xe^{\frac{1}{x}} = 0$

8. Find the locus of the centre if curveture of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

9. Find the surface area of the solid generated by the revolution of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ about the line y=0.