Ex./UG/Sc/CORE-1/MATH/TH/50/2019

BACHELOR OF SCIENCE EXAMINATION, 2019
(1st Year, 1st Semester)
MATHEMATICS(Honours)
Mathematics - I
Paper : CORE - I
(Algebra, Geometry and Calculus)

Time : Two hours | Full Marks : 50

Use a separate Answer-Script for each part.

PART - I (15 marks)
(Algebra)
Answer ***all*** questions.

1. Define orthogonal matrix.

If $(I+A)^{-1}(I-A)$

is a real orthogonal matrix prove that A is skew symmetric.

OR

show that

$$\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b2 & b^3+cda \\ 1 & c & c2 & c^3+dab \\ 1 & d & d2 & d^3+abc \end{vmatrix} = 0$$

(Turn over)

2. If

$$i^{i^{i \text{ to } \infty}} = A + iB$$

prove that

$$\tan\frac{\pi A}{2} = \frac{B}{A} \text{ and } A^2 + B^2 = e^{-\pi B}$$

[Assume only principal values].

OR

solve by Cardan's method

$$t^3 + 15t^2 - 33t - 847 = 0.$$

3. Find integers a, b, c for which

$$(x + a)(x + 1991) + 1 = (x + b)(x + c).$$

OR

Let a, b, c be positive real numbers such that

$$abc(a + b + c) = 3$$

Prove that

$$(a+b)(b+c)(c+a) \geq 8.$$

8. (a) Find the conditions for a straight line y = mx + c to be an asymptote of the curve y = f(x)

(b) Define centre and circle of curvature of the curve y = f(x). Also find the equation of circle of curvature. 2+3

9. State Leibnitz's theorem on nth order derivative of product of two functions and hence show that

$$\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = (-1)^n \frac{\lfloor n}{x^{n+1}}\left(\log x - 1 - \frac{1}{2} - \frac{1}{3} - \ldots - \frac{1}{n}\right)$$ 1+4

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PART - II (20 marks)

(Geometry)

4. Answer any ***four*** questions : 4x5=20

(a) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic

$$\frac{l}{r}=1+e\cos\theta \ \text{if} \ (l\cos\alpha-ep)^2+l^2\sin^2\alpha=p^2$$

(b) Reducing the equation $x^2+2xy+y^2-4x+8y-6=0$ to its canonical form, show that it is a parabola and determine the coordinates of its vertex.

(c) Prove that the shortest distance between the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{-1}=\frac{y-3}{3}=\frac{z-1}{2}$ is $15/\sqrt{238}$ and the equation of the line of shortest distance is $7x-37y-10z+117=0=5x+13y-17z-27$.

(d) Find the equation of the sphere which passes through the circle $x^2+y^2=4$, $z=0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3.

(e) Find the equation of a cylinder generated by the lines parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$, the guiding curve being the conic $z=0$, $ax^2+by^2=1$.

(Turn over)

(f) Show that the equation $x^2-2y^2+3z^2-4xy+5yz-6zx+8x-19y-2z-20=0$ represents a cone with vertex $(1,-2,3)$.

PART - III (15 marks)

(Calculus)

Answer any ***three*** questions.

5. (a) Show that $\int_0^{\pi/2}\cos^n x\cos nx\,dx=\frac{\pi}{2^{n+1}}\,(n\in Z_+)$.

(b) Show that the curve $y=\frac{1-x}{1+x^2}$ has three points of inflexion which lie on a straight line. 2+3

6. (a) Define hyperbolic sine of x and then show that $\cosh^2 x+\sinh^2 x=\cosh 2x$.

(b) Find the volume of the solid obtained by revolving the cardioide $r=a(1+\cos\theta)$ $(a>0)$ about the initial line. 1+4

7. (a) Find the envelope of a family of curves $F(x,y,\alpha)\equiv A(x,y)\alpha^2+B(x,y)\alpha+c(x,y)=0$, where α is a parameter.

(b) Trace the curve $x^3+y^3=3axy$ with proper justification. 2+3