for all f, g, $\in$ C and for all x $\in \mathbb{R}$. Show that $(\mathbb{R}, +, .)$ is a ring under the binary operations defined above. 5+5

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Ex/MATH/S/12/5S/2019(OLD)

BACHELOR OF SCIENCE EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

MATHEMATICS (SUBSIDIARY)

Algebra - II

Paper : 5S

Time : Two hours Full Marks : 50

Symbols and Notations have their usual meanings.

Answer any ***five*** questions.

1. (a) Solve, if possible the system of equations

$$x_1 + 2x_2 - x_3 = 10$$
$$-x_1 + x_2 + 2x_3 = 2$$
$$2x_1 + x_2 - 3x_3 = 2$$

(b) Obtain the fully reduced normal form of the matrix

$$\begin{bmatrix} 2 & -1 & 1 & 0 \\ 1 & 2 & -1 & 2 \\ 1 & -8 & 5 & -6 \end{bmatrix}$$ 5+5

2. (a) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then verify that A satisfies its own characteristic equation. Hence find A^{-1}.

(Turn Over)

(b) Investigate for what values of λ and μ the following equations.

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$, have (i) no solution (ii) a unique solution, (iii) an infinite number of solutions. 4+6

3. (a) If $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ then find the eigen values and the corresponding eigen vectors of A.

(b) If λ be an eigen value of a non-singular matrix A, then prove that λ^{-1} is an eigen values of A^{-1}. 6+4

4. (a) Expand by Laplace's method to prove that

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = \left(a^2 + b^2 + c^2 + d^2\right)^2$$

(b) Determine the values of α, β, γ such that the matrix

$\begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix}$ is orthogonal. 5+5

5. (a) For any three non empty sets A, B, C prove that

$$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$$

where $A \Delta B$ = symmetric difference of two sets A and B.

(b) Describe all the permutations on the set $\{1,2,3\}$ and find their respective orders. 4+6

6. (a) Prove that every cyclic group is an abelian group. Is an abelian group necessarily a cyclic group ? Illustrate with an example.

(b) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 7 & 5 & 2 & 3 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 6 & 7 & 3 & 5 & 2 \end{pmatrix}$

Determine whether the permutation β and α^{-1} are even or odd. 5+5

7. (a) Solve by matrix inversion method

$x + 2y + z = 4$

$x - y + z = 5$

$2x + 3y - z = 1$

(b) Let $\mathbb{R}$ be the set of real numbers and C be the set of all real valued continuous functions defined on $\mathbb{R}$. Define $(f+g)(x) = f(x) + g(x)$ $(f.g)(x) = f(x) . g(x)$,

(Turn Over)