(b) Solve $x^7 = 1$ and hence find the sum of 99th plowers of the roofs of the equation $x^7 - 1 = 0$ 3+2

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Ex/MATH/H/12/2.3/54/2019(OLD)

BACHELOR OF SCIENCE EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

MATHEMATICS (HONOURS)

Algebra - II

Paper : 2.3

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.
Symbols and Notations have their usual meanings.

Answer any ***five*** questions.

1. (a) (i) Define binary relation and binary operation on a nonempty set. Let S be a set with n elements. Find the number of binary relations and the number of binary operations defined on S. 3

(ii) Find the order of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 9 & 6 & 3 & 2 & 1 & 8 & 7 & 5 \end{pmatrix}$$ 2

(b) (i) Let a and b be two integers and p be a prime such that $p|ab$. Show that $p|a$ or $p|b$.

(ii) Show that for any positive integer n; $n(n+1)(n+2)$ is divisible by 3. 2+3

2. (a) Define group and subgroup of a group. Is union of two subgroups of a group also a subgroup of that group ? Justify your answer. 2+1+2

(b) (i) Let G be a group such that every element of G has its own inverse. Show that G is a commutative group.

(ii) Let a and b be two elements of a group G such that $0(a)=5$ and $a^3b=ba^3$. Show that $ab=ba$. 2+3

3. (a) Define a cyclic group. Let G be a group and z(G) be the centre of G. If G/z(G) is a cyclic group then show that G is a commutative group. If G is a non commutative group then show that $|Z(G)|\le\frac{1}{4}|G|$. 1+2+2

(b) State and prove Lagrange's theorem on group. 2+3

4. (a) State first isomorphism theorem of group. Show that

$$GL(n,\mathbb{R})/SL(n,\mathbb{R}) \cong \mathbb{R}^*$$ 2+3

(b) (i) When is a subgroup H of a group G said to be normal subgroup of G ? Show that Z(G) is a normal subgroup of G. 1+2

(ii) Let H and K be two normal subgroups of a group G such that $H\cap K=\{e_G\}$. Show that $ab=ba$ for all $a\in H$ and $b\in K$. 2

5. (a) Define integral domain. Is C[0,1] an integral domain ? Justify your answer. 2+3

(b) Define ideal of a ring. Give an example of a left ideal in a ring R which is not a right ideal of R. Let I be an ideal of a ring R with unity such that $1\in I$. Show that $I=R$. 2+2+1

6. (a) Define Boolean ring. Show that a Boolean ring has characteristic 2 and also show that every Boolean ring is commutative. 1+2+2

(b) Define division ring. Show that the centre of a division ring is a field. 2+3

7. (a) (i) Show that the different values of i^i are all real and they form a G. P.

(ii) Let n be a positive integer. Show that

$$(1+i)^n+(1-i)^n=2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$$ 2+3

(Turn Over)