Ref. No.: EX/MATH/S/12/6S/55/2019(OLD)

Bachelor of Science Examination, 2019 (OLD)

(1st Year, 2nd Semester)

Mathematics

Paper-6S

(Vector Algebra)

Full Marks: 50 Time: Two Hours

Answer any five questions.

(Use vector approach to solve all problems)
($\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors in the positive directions of x, y and z axis respectively)

1. (a) Suppose ABCD is a parallelogram. P,Q are the midpoints of the sides AB and CD respectively; show that DP and BQ trisect AC and are trisected by AC. (4)

   (b) ABC is a triangle. D divides BC in the ratio $l : m$; G divides AD in the ratio $l + m : n$. Find the position vectors of D and G. (2)

   (c) Show that the four points whose position vectors are given by $-6\vec{a}+3\vec{b}+2\vec{c}$, $3\vec{a}-2\vec{b}+4\vec{c}$, $5\vec{a}+7\vec{b}+3\vec{c}$ and $-13\vec{a}+17\vec{b}-\vec{c}$ are coplanar; $\vec{a}, \vec{b}, \vec{c}$ being three non-coplanar vectors. (4)

2. (a) Prove that medians of a triangle are concurrent. (4)

   (b) Find a unit vector parallel to the resultant of vectors $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$. (2)

   (c) Prove that a necessary and sufficient condition that the vectors $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$, $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$ and $\vec{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$ be lineraly independent is that the determinant (4)

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

   be different from zero.

3. (a) Find the projection of the vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$. (2)

(b) Prove that an angle inscribed in a semi-circle is a right angle. (3)

(c) Two sides of a triangle are formed by the vectors $\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{B} = 4\hat{i} - \hat{j} + 3\hat{k}$. Determine all angles of the triangle. (5)

4. (a) Derive the expression for scalar product of four vectors. Hence prove that (3)

$$(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) + (\vec{B} \times \vec{C}) \cdot (\vec{A} \times \vec{D}) + (\vec{C} \times \vec{A}) \cdot (\vec{B} \times \vec{D}) = 0.$$

(b) Prove that $\sin(A - B) = \sin A \cos B - \cos A \sin B$. (3)

(c) Find the constant a such that the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar. (2)

(d) A force given by $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at a point $(1, -1, 2)$. Find the moment of $\vec{F}$ about the point $(2, -1, 3)$. (2)

5. (a) Use vector algebra to find the equation of the plane passing through the point (2,3,-1) and perpendicular to the vector (3,-4,7). Write the equation in cartesian form. Find the length of the perpendicular from the origin to the plane. (5)

(b) Find the equation of the plane containing the line $\vec{r} = \vec{a} + t\vec{b}$ and perpendicular to the plane $\vec{r}.\vec{c} = q$. (5)

6. (a) Find the vector equation of the line of intersection of the planes $3x - y + z = 1$ and $x + 4y - 2z = 2$. (5)

(b) Find the shortest distance between two lines, one joining the points $A(-1, 2, -3)$ and $B(-16, 6, 4)$ and the other joining the points $C(1, -1, 3)$ and $D(4, 9, 7)$. (5)

7. (a) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 49$ at the point $(6, -3, -2)$. Show further that $2x - 6y + 3z - 49 = 0$ is a tangent plane to the same sphere. (5)

(b) Derive the condition for which the plane $\vec{r}.\vec{n} = p$ should touch the sphere $\vec{r}.\vec{r} - 2\vec{r}.\vec{c} + k = 0$ (5)