Bachelor of Science Examination, 2019 (OLD)

(1st Year, 2nd Semester)

STATISTICS (Subsidiary)

Paper: 5-Stat

Full Marks : 50 Time : Two hours

The figures in the margin indicate full marks.

(Symbols/ Notations have their usual meanings)

Answer ***question No.6 and any three*** *from the rest.*

1.(a) Let X_1 and X_2 be two independent random variables having Geometric distribution $q^k p$, $k = 0,1,2,\ldots$. Find the conditional distribution of X_1 given that $X_1 + X_2 = n$.

(b) If $X \sim N(\mu,\sigma^2)$ then obtain the p.d.f. of $U = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$.

(c) If X has a uniform distribution in [0, 1], find the distribution (pdf) of $-2\log X$.

(d) If $X \sim N(\mu,\sigma^2)$ then show that mean deviation about mean is $\frac{4}{5}\sigma$.

4+4+4+4

2.(a) Suppose that the probability density function of a random variable X is given by

$$f(x) = \frac{x^{\lambda-1}e^{-x}}{\Gamma(\lambda)}, \quad for\ x>0,\ \lambda>0$$
$$= 0, \quad otherwise$$

(i) Find the moment generating function of this distribution.

(ii) Determine the mean and variance of this distribution, using moment generating function.

(b) If X is uniformly distributed in the interval (-1, 1), find the distribution of $|X|$.

(c) If X is a $\beta_2(l,m)$ variate then show that $Y = 1/X$ is a $\beta_2(m,l)$ variate.

(d) The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a Gamma variate with parameters $\alpha = \frac{1}{10{,}000}$ and $\lambda = 2$. The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day?

6+3+3+4

3.(a) The joint probability distribution function of two random variables X and Y is given by

P(X = 0, Y = 1) = 1/3

P(X = 1, Y = -1) =1/3

P(X = 1, Y = 1) = 1/3

Find (i) marginal distributions of X and Y.

(ii)conditional distribution of X given Y = 1.

(b) For a normal $N(\mu,\sigma^2)$ distribution, show that

$$\mu_{2n+1} = 0,\ n=0,1,2,\ldots\ldots$$
$$\mu_{2n} = 1.3.5\ldots\ldots(2n-1)\sigma^{2n}$$

(c) A man always carries two match boxes (initially containing N match sticks). Each time he wants a match stick, he selects a box at random. Ultimately a moment comes when he finds a box is empty. Find the probability that there are exactly r match sticks in one box when the other box is empty.

6+6+4

4.(a) The joint probability density function of two random variables X and Y is given by

$$f(x,y) = x+y,\quad 0<x<1,\ 0<y<1$$
$$=0,\ elsewhere$$

Find the distribution of XY.

(b) If X and Y are two independent normal variates (m_x,σ_x) and (m_y,σ_y), respectively then $U=X+Y$ is a normal variate (m,σ) where $m=m_x+m_y$ and $\sigma^2=\sigma_x^2+\sigma_y^2$.

(c) Two numbers are independently chosen at random between 0 and 1. Show that the probability that their product is less than a constant k $(0<k<1)$ is $k(1-\log k)$.

6+6+4

5.(a) If X be a continuous random variable with $E(X) = \mu$ and $Var(X) = \sigma^2$ then show that for $\varepsilon>0,\ P(|X-\mu|\geq\varepsilon)\leq\frac{\sigma^2}{\varepsilon^2}$.

(b) Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with mean μ and variance $\sigma^2(>0)$. Prove that $\frac{S_n-n\mu}{\sigma\sqrt{n}} \xrightarrow{p} N(0,1)$ where $S_n = X_1+X_2+\ldots+X_n$.

(c) Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 inch and a standard deviation of 1.0 inch.

(i) If one male is randomly selected, find the probability that his head breadth is less than 6.2 inch.

(ii) Find the probability that 100 randomly selected men have a mean head breadth that is less than 6.2 inch.

Given that the area under the standard normal curve between z = 0 to z = 0.2 is 0.0793 and the area between z = 0 to z = 2 is 0.4773.

6+6+4

6. State weak law of large numbers. 2