b) Solve by Laplace Transform method :

$$\frac{d^2y}{dt^2} + y = 6\cos 2t,\ y(0) = 3,\ \frac{dy}{dt} = 1 \ \text{ when } t = 0.$$

14. a) Find inverse Z-transform of

$$\frac{Z^3 - 20Z}{(Z-2)^3(Z-4)}$$ 4

b) Solve the difference equation by Z-transform :

$$u_{n+2} + 6u_{n+1} = 9u_n = 2^n$$

with $u_0 = u_1 = 0.$ 6

BACHELOR OF ENGINEERING EXAMINATION, 2019

(1st Year, 2nd Semester)

MATHEMATICS - II

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use a separate Answer-Script for each part

PART I (50 Marks)

Answer ***any five*** questions. 10×5=50

1. a) What is meant by an a lternating series ? Give an examples.

b) Test the convergence of any two of the following series-

i) $\frac{1+2}{2^3} + \frac{1+2+3}{3^2} + \frac{1+2+3+4}{4^3} + \ldots\ldots\ldots.$

ii) $1 + \frac{1}{L1} + \frac{2^2}{L2} + \frac{3^2}{L3}$

iii) $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \ldots..$

c) Find the radius of covvergence of any one of the following prower series

i) $x + \frac{2^2x^2}{21} + \frac{3^3x^3}{31} + \ldots\ldots\ldots.$

ii) $x + \frac{(L2)^2}{L4}x^2 + \frac{(L3)^2}{L6}x^3 + \ldots\ldots\ldots$ 1+(3+3)+

[Turn over

2. a) State any one term of the Fundamental Theosem of Integral Calculus.

   b) State any one torm of the Mean value Theorem of integral calculus.

   c) Let I, m, n - $\int Sin^m x \cos^n xdx$, Deduce the reduction formula $I m, n = \frac{Sin^{m-1}xCos^{n+1}x}{m+n} + \frac{m-1}{m+n} I_{m-2}, n$

   d) Deduce the following rediction formula for the crammafuction. () 2+2+3+3

3. a) Why is Beta finction an improper integral ? Discuss its cpnvergence.

   b) Usnog cpmpanison test or otherwise test the convergence of the improper integral $\int_D^\alpha \frac{dx}{e^x+1}$

   c) Let F(x)= $\int_0^1 \log\left(x^2+y^2\right)dy$ Find $F^1(x)$ 5+2+3

4. a) Using double integration find the area enclosed by the paabolas $y^2=4x$ and $x^2=4y$.

   b) Using triple integration find the volume of the sphere $x^2+y^2+z^2=16$ 4+6

5. a) Change the order of integration is

   (i) $\int_0^1 \int_{x^3}^{x^2} f(x,y)dydx$ ii) $\int_0^1 \int_{x^2}^{2-x} f(x,y)dydx$

   b) Evaluate $\iint_R x^{m-1}y^{n-1}dxsy$ in terma of Gamma funetion whese m, n>0 and R is the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

6. a) Find the mass of the tetrahedron bounded by the co-ordinte planes and the plane $\frac{x}{2}+\frac{y}{3}+\frac{z}{5}=1$ Where the density $P=Mx^2y^2z^2$

   b) Find the volume enclosed by $f(x,y)=x^2y^2$ and the planes x=0, x=3, y=-1, and y=1

   c) Evaluate $\int_0^\alpha e^{-t}t^{\frac{5}{2}}dt$ 5+3+2

7. a) Show that the sequance $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2}+\sqrt{2}}$ is convergent and find its limit.

   b) Prove that every monotonic increasing sequence which is dounded above is convergent.

   Does it remain bue if the sequnce is founded below istead of bounded above ? 4+4+2

[Turn over

PART II (50 Marks)

Answer ***any five*** questions. 10×5=50

Symbols / Notations have their usual meanings.

8. a) If a function $f(z) = u(x, y) + iv(x, y)$ is differentiable at a point $z = x + iy$, prove that it satisfies Cauchy-Riemann equations. 5

b) Show that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and determine its harmonic conjugate. Hence find the corresponding analytic function $f(z)$. 5

9. a) Prove that the function $f(z)$ defined by

$$f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, \ (z \neq 0), f(0) = 0$$

is continuous and satisfies Cauchy-Riemann equation at the origin, yet $f'(0)$ does not exit. 6

b) If $f(z)$ is a regular function of z, prone that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2 \qquad 4$$

10. a) Expand $f(z) = \frac{1}{(2+1)(2+3)}$ in Laurent series valid for the region $1 < |z| < 3$. 5

b) Evalute the integral

$$\int_C \frac{dz}{z^2(z+1)(z-1)}, \ C: |z = 3| \qquad 5$$

11. Find the Fourier series for the function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 1 - x, & 1 < x < 2 \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

12. a) Find Fourier transfrom of f(x) defined by

$$f(x) = \begin{cases} 1, & |x| \leq a \\ 0, & |x| > a \end{cases}$$

where a is a positive real number, and hence evaluate

$$\int_0^\infty \frac{\sin x}{x} dn \qquad 5$$

b) Using convolution theorem of Fourier transform, prove that

$$\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)} = \frac{\pi}{2ab(a+b)}. \ (a, b \neq 0)$$

5

13. a) Find the inverse Laplace transform of

$$\frac{3s+1}{s^2(s^2+4)} e^{-3s}$$