10. Determine the nature of solution $x = \infty$ at of the differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 12y = 0.$$

Find series solution at infinity of the above equation. Write first three non-zero terms in each infinite series. 9

11. Prove that

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & , m \neq n \\ \dfrac{2}{2n+1} & , m = n \end{cases}$$

where $P_x(x)$ is the Legendre polynomial of degree n. 7

FIRST B.SC. EXAMINATION, 2019

(1st Year, 2nd Semester)

MATHEMATICS

CORE - 4

(DIFFERENTIAL EQUATION)

Time : Two hours Full Marks : 50

(25 marks for each part)

Use a separate Answer-Script for each part

PART - I

(Answer ***any five*** questions) 5×5=25

Symbols and notations have their usual meaning.

All questions are of equal marks.

1. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^x \cos 2x + \cos 3x.$

2. Solve $(x+2)^2\frac{d^2y}{dx^2} - 4(x+2)\frac{dy}{dx} + 6y = x.$

3. Use the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x.$$

4. Reduce to Clairaut's form and solve the following :

$$xy(y - px) = x + py.$$

5. Find the singular solution of

$$xp^2 - 2yp + ax = 0.$$

6. Does unique solution of $\frac{dy}{dx} = y/x,\ y(0) = 0,$ exist ? Justify your answer.

7. Answer ONE of the following :

   a) Find the orthogonal trajectories of $y^2 = 4ax.$

   b) Use the method of undetermined coefficient to solve

$$\frac{d^2y}{dx^2} + 4y = \sin 3x.$$

PART - II

(25 marks)

(Answer Question number ***11*** and ***two*** questions from the rest.)

8. Find two solutions of the system of equations

$$\frac{dx}{dt} = x - 4y\ ;\ \frac{dy}{dt} = x + y,$$

using eigenvalues and eigenvectors of the associated matrix. Find the Wronskian of the two solutions. Hence obtain the general solution. Verify that the general solution satisfies the given system of equations. Find a particular integral satisfying $x(0) = 2$ and $y(0) = 1.$ 9

9. a) Use operator method to solve the system of equations given by

$$2\frac{dx}{dt} + 4\frac{dy}{dt} + x - y = 3e^t\ ;\ \frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = e^t.$$ 5

   b) Use series method to obtain general solution of the Cauchy-Euler equation

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + \frac{3}{4}y = 0.$$ 4