b) Find the equation of the plane which contains the straight line $x = \frac{y-3}{2} = \frac{z-5}{3}$ and perpendicular to the plane $2x + 7y - 3z = 1$. 4+6

8. a) Find the shortest distance between the straight lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-5}{4} = \frac{y-4}{4} = \frac{z-5}{5}.$$

Hence show that the lines are coplanar.

b) Find the equation of the sphere described on the join of P(2, –3, 4) and Q(–5, 6, –7) as diameter. 7+3

9. Find the centre and the radius of the circle

$$x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$$

and $x + 2y + 2z = 15$. 10

First B. Sc. Examination, 2019

(1st year, 1st Semester, Old Syllabus)

Mathematics

(Analytical Geometry)

Paper : 3S

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.

Group - A (20 marks)

Answer ***any two*** questions. 2×10

1. a) Prove that a homogeneous second degree equation always represents a pair of straight lines passing through the origin.

b) Find the condition that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ may coincide with one of the straight lines given by $a'x^2 + 2h'xy + b'y^2 = 0$. 5+5

2. Reduce the equation $x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$ to its canonical form and determine the nature of the conics. Hence find the centre, eccentricity if any. 10

3. a) If a pair of diameters be conjugate with respect to a hyperbola, then they are also conjugate with respect to its conjugate hyperbola.

b) Find the equation of the diameter of the ellipse $3x^2 + 4y^2 = 5$ conjugate to the diameter $y + 3x = 0$. 6+4

4. a) Show that the following circles cut each other orthogonally :

$$x^2 + y^2 - 2bx + c = 0$$

and $x^2 + y^2 + 2ay - c = 0$

b) Find the radical axis of the two circles $x^2 + y^2 + 4x - 2y + 9 = 0$ and $x^2 + y^2 + 2x + 3y - 5 = 0$

c) Find the limiting points of the co-axial system defined by the pair of circles $x^2 + y^2 - 6x - 8y + 5 = 0$ and $x^2 + y^2 - 8x - 10y + 5 = 0$. 3+2+5

Group - B (30 marks)

Answer ***any three*** questions. 3×10

5. a) The projections of a line segment on the axes are 3, 4, 12. Find the length and the direction cosines of the line.

b) If (l_1, m_1, n_1), (l_2, m_2, n_2) be the direction cosines of two mutually perpendicular straight lines, then show that the direction cosines of the straight line perpendicular to both of them are $\pm(m_1n_2 - m_2n_1)$, $\pm(n_1l_2 - n_2l_1)$ and $\pm(l_1m_2 - l_2m_1)$. 4+6

6. a) Find the value of h for which the planes $3x - 2y + hz - 1 = 0$ and $x + hy + 5z + 2 = 0$ may be perpendicular to each other.

b) Find the angle between the planes $x - y + 2z = 9$ and $2x + y + z = 7$.

c) A variable plane which is at a constant distance 3p from the origin O cuts the axes in A, B, C. Show that the locus of the centroid of the triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$. 2+2+6

7. a) Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$.