First B. Sc. Examination, 2019

(1st year, 1st Semester, Old Syllabus)

Mathematics

Paper : IS (Calculus)

Time : Two hours | Full Marks : 50

Symbols / Notations have their usual meaning.

Answer ***any five*** questions.

1. a) State mean value theorem in Lagrange's form.

Let f be real valued function defined over [–1, 1] such that

$$f(x) = \begin{cases} x\cos\frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

Does the mean value theorem hold for f in [–1, 1] ?

b) Use mean value theorem of appropriate order to prove that

$\sin x > x - \frac{x^3}{3!}$, when $0 < x < \pi/2$. 5+5

2. a) State Leibnitz's theorem on successive - Differentiation. Using this theorem prove that if $y = \cos(m\sin^{-1}x)$, then

$$(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} + (m^2 - n^2)y_n = 0$$

Also find y_n for x = 0. 4+6

[Turn over

3. a) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$

b) If $\phi(x)$ be a polynomial in x and λ be a real number, then prove that there exists a root of $\phi'(x)+\lambda\phi(x)=0$ between any pair of roots of $\phi(x)=0$. 5+5

4. a) Show that the semi-vertical angle of the right cone of given total surface (including area of the base) and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$, when the volune is Maximum.

b) Show that the minimum value of $\frac{(2x-1)(x-8)}{x^2-5x+4}$ is greater that its local maximum value. 5+5

5. a) Find the asymptotes of the curve

$$y^3-6xy^2+11x^2y-6x^3+y^2-x^2+2x-3y-1=0$$

b) Find the radius of curvature at the origin of

$$y^2=\frac{x^2(a+x)}{(a-x)}$$ 5+5

6. a) Test the convergence of the following :

i) $\int_1^\infty \frac{dx}{x(1+x)}$

ii) $\int_1^\pi \frac{dx}{1+\cos x}$ 5+5

b) Define Beta function. Prove that B(m, n) is convergent for $m>0$ and $n>0$.

7. a) Prove that $B(x,y)=2\int_0^{\pi/2}\sin^{2x-1}\theta\cos^{2x-1}\theta\, d\theta$,

Also prove that $B\left(\frac{1}{2},\frac{1}{2}\right)=\pi$.

b) Evaluate $\iint_R \sin(x+y)\,dx\,dy$

over $R:\begin{cases}0\le x\le \pi/2\\ 0\le y\le \pi/2\end{cases}$ 6+4