First B. Sc. Examination, 2019

(1st year, 1st Semester, Old Syllabus)

Mathematics (Subsidiary)

Paper : IIS (Algebra - I)

Time : Two hours | Full Marks : 50

(Notations and symbols have their usual meaning)

Answer ***any five*** questions.

1. a) If z is a complex variable such that mod $\left(\frac{z-i}{z+1}\right)=k$, show that the point z lies on a circle in the complex plane if $k \neq 1$. Also find the radius and centre of the circle.

   b) Define Log z and log z, where Z is a non-zero complex number.

   If z_1, z_2 are two distinct complex numbers such that $z_1 z_2 \neq 0$, then show that $\text{Log} z_1 + \text{Log} z_2 = \text{Log}(z_1 z_2)$.

   Is $\log z_1 + \log z_2$ equal to $\log(z_1 z_2)$? Justify. 5+5

2. a) If α, β are the roots of the equation $t^2 - 2t + 5 = 0$ and n is a positive integer, prove that

   $$\frac{(a+\alpha)^n - (a+\beta)^n}{\alpha - \beta} = 2^{n-1} \sin n\phi \operatorname{cosec}^n \phi,$$ where 'a' is a real number satisfying $\frac{a+1}{2} = \cot\phi$.

[Turn over

b) If a, b, c d > 0 and a + b + c + d = 1, prove that

$$\frac{a}{1+b+c+d}+\frac{b}{1+a+c+d}+\frac{c}{1+a+b+d}+\frac{d}{1+a+b+c}\geq\frac{4}{7}$$

5+5

3. a) Using the property of complex numbers, show that $a^3\cos 3B + 3a^2 b\cos(2B - A) + 3ab^2\cos(B - 2A) + b^3\cos 3A = c^3$, where a, b, c , A, B, C have their usual meanings in a triangle ABC.

b) If x is a real number then prove that

$$i\log\left(\frac{x-i}{x+i}\right)=\begin{cases}\pi-2\tan^{-1}(x) & \text{if} \quad x>0\\ -\pi-2\tan^{-1}(x) & \text{if} \quad x\leq 0\end{cases}$$

5+5

4. a) Define limit of a sequence.

Show that the sequence $\{x_n\}$, where

$$x_n=\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdot\cdot+\frac{1}{(2n+1)(2n+3)}$$ is monotone.

b) Show that the sequence $\{y_n\}$, where

$$y_n=1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}$$

is monotone increasing sequence and bounded above. Is the sequence $\{y_n\}$ convergent ? – Justify. 5+5

5. a) State and prove Raabe's test for convergence or divergence of a series of positive terms.

b) Test the convergence of the following infinite series

$$\sin(1)+\frac{1}{2}\sin\left(\frac{1}{2}\right)+\frac{1}{3}\sin\left(\frac{1}{3}\right)+\cdots$$

5+5

6. a) Write the Descarte's rule of signs. Apply the rule to examine the nature of roots of the euqtion $x^6+x^4+x^2+x+3=0$

b) Solve $x^3-3x-1=0$ by cardan's method. 5+5

7. a) Solve the equation $16x^4-64x^3+56x^2+16x-15=0$ where roots are in arithmetic progression.

b) If a.b,c are positive real numbers, not all euqal, prove that

$$2(a^3+b^3+c^3)>a^2(b+c)+b^2(c+a)+c^2(a+b)>6abc$$

5+5