BACHELOR OF SCIENCE EXAMINATION, 2019

(1st Year, 2nd Semester, Old Syllabus)

MATHEMATICS (HONOURS)

Differential Equation - I

Paper : 2.2

Time : Two hours Full Marks : 50

Symbols and Notations have their usual meanings.

Answer any ***five*** questions.

1. (a) Find one solution of a second order linear differential equation $y'' + p(x)y' + q(x)y = X(x)$ by charging the independent variable x to z such that $z = f(x)$.

   (b) Find general and singular solution of the differential equation $y = 2\left(\frac{dy}{dx}\right)x + \left(\frac{dy}{dx}\right)^2$. 5+5

2. (a) Solve the system of equation $\frac{dx}{dt} + \frac{dy}{dt} + 2y = 0$

$$\frac{dx}{dt} - 3x - 2y = 0$$

(Turn Over)

(b) Reduce the differential equation

$$(x+2)^2\frac{d^2y}{dx^2}+(x+2)\frac{dy}{dx}+4y=2\,sin\{2\log(x+2)\}$$

to Euler-Cauchy form and hence solve it. 5+5

3. (a) Reduce the differential equation

$$\sin y\frac{dy}{dx}=\cos x\left(2\cos y-\sin^2 x\right)$$

to a linear form and hence solve it.

(b) Solve : $x^2y'' + xy' + y = \log x \,.\, \sin(\log x)$ 5+5

4. (a) Define integrating factor of a differential equation M(x,y)dx + N(x,y)dy = 0.

If an integrating factor does exist, is it unique ? Justify your answer.

(b) Show that the solution of $\frac{dy}{dx}+P(x)y=\phi(x)$ can be written in the form

$$y=\frac{\phi}{P}-e^{-\int Pdx}\left[c+\int e^{\int Pdx}\,d\left(\frac{\phi}{P}\right)\right]$$

(C is an arbitrary constant).

(c) State a sufficient condition for existence of the solution of $\frac{dy}{dx}=f(x,y)$. (1+3)+5+1

5. (a) Show that $\frac{1}{F(D)}\left(e^{mx}V(x)\right)=e^{mx}\frac{1}{F(D+m)}V(x)$

(b) Find the orthogonal trajectories of $\frac{x^2}{a^2}+\frac{b^2}{b^2+\lambda}=1$, where λ is a parameter. 6+4

6. (a) Find particular integral of $(D^2-1)y = x^2\cos x$.

(b) Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+a^2y=\tan ax$. 5+5

7. (a) Solve : $\frac{dx}{mz-ny}=\frac{dy}{nx=\ell z}=\frac{dz}{\ell y-mx}$

(b) Solve : $x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1$

(c) State a necessary and sufficient condition for a differential equation of first order and first degree to be exact. 5+4+1