7. Show that the equation of the right circular cylinder whose guiding curve is the circle through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) is $x^2 + y^2 + z^2 - yz - zx - xy = 1$.

Ex/IM/II/12/19(Old)

First B.Sc. Examination, 2019

(1st Year, 1st Semester, Old Syllabus)

Mathematics (Honours)

Paper - 1.2

(Geometry)

Time : Two hours

Full Marks : 50

Use a separate answerscript for each Part.

Part -I (20 Marks)

Answer any two questions

1. a) Show that the necessary and sufficient conditions that the general equation of second degree in x,y should represent a pair of straight lines is

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$$

b) Prove that the transformation of rectangular axes which converts $\frac{x^2}{p} + \frac{y^2}{q}$ into $ax^2 + 2hxy + by^2$, will convert $\frac{x^2}{p-\lambda} + \frac{y^2}{q-\lambda}$ to

$$\frac{ax^2 + 2hxy + by^2 - \lambda\left(ab - h^2\right)\left(x^2 + y^2\right)}{1 - (a+b)\lambda + \left(ab - h^2\right)\lambda^2}$$

6+4

[Turn over

2. a) Show that the straight lines joining the origin to the points of intersection of the curves $ax^2 + 2hxy + by^2 + 2gx = 0$ and $a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$ will be at right angle if $g(a' + b') = g'(a + b)$.

b) If the equation of the conic be $ax^2 + 2hxy + by^2 + c = 0$, show that origin is the centre.

c) Reduce the equation $x^2 - 6xy + 9y^2 + 4x - 12y + 4 = 0$ to the canonical form and determine the type of the conic represented by it. 3+2+5

3. a) Find the general equation of conics confocal with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

b) Prove that equation to the hyperbola, passing through the points on the ellipse whose eccentric angle is α and which is confocal with the ellipse, is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = a^2 - b^2$

c) If PSP' and QSQ' be any two focal clords of a conic which are at right angles to one another, prove that $\frac{1}{SP.SP'} + \frac{1}{SQ.SQ'} =$ a constant. Where S is the focus.

2+4+4

Part - II (30 marks)

Answer ***any six*** questions 6× 5=30

1. A variable plane passes through a fixed point (α, β, γ) and meets the axes of reference in A, B, C show that the locus of the point of intersection of the planes through A, B, and C parallel to the coordinate plane is $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 1$.

2. Show that the four points (0, –1, 0), (2, 1, –1), (1, 1, 1) and (3, 3, 0) are coplanar and obtain the equation of the plane.

3. Prove that the distance of the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$ from the point (–1, –5, –10) is 13.

4. Find the equations of the image of the line $\frac{x-2}{2} = \frac{y-3}{3} = \frac{z-4}{4}$ in the plane $3x + y - 4z + 21 = 0$.

5. Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point $(2, -4, 6)$.

6. Find the locus of the vertices of the right circular cones that passes through the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$.

[Turn over