Ref. No.: **Ex/Prod/Math/T/211/2019 (OLD)**

BACHELOR OF PRODUCTION ENGINEERING EXAMINATION – 2019 (OLD)

(2ND YEAR 1ST SEMESTER, SUPPLEMENTARY, OLD)

MATHEMATICS – VS

FULL MARKS: 100 TIME: 3 HOURS

Answer any *six* questions.
Four marks are reserved for neatness.
(Notations have their usual meanings)

1. (a) Prove that the function $u = e^{-2xy}\text{Sin}(x^2 - y^2)$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Express f(z) in term of z.

   (b) Determine the analytic function whose real part is $\sin 2x/(\cosh 2y - \cos 2x)$. 9+6

2. (a) Show that the polar form as Cauchy-Riemann equations are

   $$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{d\theta}, \frac{\partial v}{dr} = -\frac{1}{r}\frac{\partial u}{d\theta}$$

   Hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$

   (b) Find the analytic function whose real part is $y + e^x \cos y$ 10+6

3. (a) State and prove Liouville's theorem.

   (b) Show that the function $f(z) = \cos 2z$ satisfies Cauchy-Riemann equations. 9+7

4. (a) Prove that if f(z) is analytic in a region R and on its boundary C, then

   $$\oint_C f(Z)\, dz = 0.$$

   (b) Evaluate $\oint_C \frac{Cosz}{z^3+z}\, dz$ *over the circle* $|z| = 5$ which is C

   (c) Evaluate $\int_0^{1+i}(x^2 + iy)dz$ along the paths $y = x$ and $y = x^2$. 5+6+5

5. (a) Show that for a Legendre function $P_n(x)$

   $$(1 - 2xh + h^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} h^n P_n(x)$$

   (b) Prove that $(2n + 1)x\, P_n\,(x) = (n + 1)P_{n+1}(x) + nP_{n-1}(x)$. 10+6

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6. (a) Solve $\frac{d^2y}{dx^2}$+(x-1)$\frac{dy}{dx} + y = 0$, in powers of x.

   (b) Express f(x) $= 4x^3 + 6x^2 + 7x + 2$, in terms of Legendre polynomials. 9+7

7. (a) Prove that

$$J_n(x) = \frac{1}{\pi} \int_0^{\pi} cos(n\theta - xsin\theta)d\theta$$

   (b) Prove that

$$J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$$ 10+6

8. (a) Show that for a Legendre function $P_n(x)$

$$(1 - 2xh + h^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} h^n P_n(x)$$

   (b) Prove that $x\, P_n'(x) = P_{n-1}'(x) + nP_n(x)$. 10+6