Bachelor of Engineering in Production Engineering Examination, 2019

(1st Year, 1st Semester, Old)

Mathematics - IN

Time : Three hours Full Marks : 100

Answer ***questions no. 1*** and ***any six*** from the rest.

1. Define a homogeneous function and determine the degrees of the following functions :

   i) $x\sin\frac{y}{x} + y\cos^{y/x}$

   ii) $\frac{x^{1/2} + y^{1/2}}{y^{1/3} - x^{1/3}}$ 2+1+1

2. a) Find the n^{th} derivative of $\frac{1}{x^2 + a^2}$ and simplify if with the help of DeMoivre's theorem. 4

   b) If $y = e^{a\sin^{-1}x}$ then apply Leibnitz theorem to prove that

   $$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

   Hence find y_n at $x = 0$ 8+4

3. a) A cylindrical tin cone closed at both ends of a given capacity has to be constructed. Show that the amount of the tin required will be minimum when the height is equal to the diameter.

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b) Find the extreme values of the function

$$f(x,y) = 4x^2 - xy + 4y^2 + x^3y + xy^3 - 4$$

4. a) If $v = \tan^{-1}\dfrac{x^3+y^3}{x+y}$, then show by Euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

b) If $v = \log_e(x^3 + y^3 + z^3 - 3xyz)$, prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

c) If $u = f(r)$ where $r^2 = x^2 + y^2$, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f'(r) + \frac{1}{r}f'(r)$$

5. a) Define Gamma function. Show that

$$\overline{|(n)} = 2\int_0^\infty e^{-y^2 y^{2n-1} dy}$$

b) Prove that

$$\iint_D x^{l-1}y^{m-1}dydx = \frac{\overline{|(l)}\,\overline{|(m)}\,a^{l+m}}{\overline{|(l+m+1)}}$$

where D is the domain $x \ge 0, y \ge 0$ and $x + y \le a$.

c) Show that

$$\frac{\overline{|n+\frac{1}{2}}}{\overline{|(n+1)}} = \frac{1,3,5\cdots(2n-1)\sqrt{\pi}}{2^n \underline{|n}}$$

6. a) Evaluate the following integrals :

i) $\displaystyle\int_{\pi/4}^{3\pi/4} \frac{\text{Sinxdx}}{\text{Cosx} - 5\text{Cosx} + 4}$

ii) $\displaystyle\int_0^{\sqrt{2}} x^3 e^{x^2} dx$ 4+4

b) With the help of Beta function and Gamma function show that

$$\int_0^{\pi/2} \text{Sin}^m x dx \text{ X } \int_0^{\pi/2} \text{Sin}^{m+1} x dx = \frac{\pi}{2(p+1)}, p > -1$$ 8

7. a) Find the area bounded by the parabola $x = 4 - y^2$ and the y axis. 8

b) The position of a point at any time t is given by $x = \frac{1}{2}t^2$, $y = \frac{1}{9}(6t+9)^{3/2}$ find the distance that the point panels from $t = 0$ to $t = 4$. 8