12. a) Expand sinhx in an infinite series in powers of x.

b) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$. 5+5

13. a) If $u = f(x^2 + 2yz, y^2 + 2zx)$, prove that

$$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0.$$

b) If $u = \cos^{-1}\{(x+y)/(\sqrt{x}+\sqrt{y})\}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$ 5+5

14. a) Find the maximum and minimum values of

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$

b) If $y = x^{n-1} l\, n\, x$, then show that

$$y_n = \frac{(n-1)!}{x}.$$ 5+5

Ex/Prod/Math/T/113/2019(Old)

Bachelor of Engineering in Production Engineering Examination, 2019

(1st Year, 1st Semester, Old)

Mathematics - I

Time : Three hours | Full Marks : 100

(50 marks for each Part)

Use a separate Answer-Script for each Part

PART - I

(Symbols/Notations have their usual meanings)

Answer ***any five*** questions.

1. a) Determine the conditions for which the system

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + az = b^2$$

admits of (i) only one solution, (ii) no solution, (iii) many solutions.

b) Use Laplace method to prove the identity

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2 + b^2 + c^2 + d^2)^2.$$ 5+5

[Turn over

2. a) State Caley-Hamilton theorem and use it to find A^{100}, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

b) If λ is an eigenvalue of A, where A is a real orthogonal matrix, then prove that $\frac{1}{\lambda}$ is also an eigenvalue of A.

5+5

3. a) Obtain the row reduced Echelon form of the matrix

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{bmatrix}.$$

b) Prove that the eigenvalues of a real skew symmetric matrix are purely imaginary or zero. 5+5

4. a) Show that, in the pormetric curve

$x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, $y = a\sin t$, the portion of the tangent intercepted between the curve and the x-axis is of constant length.

PART - II

Answer ***any five*** questions.

8. a) Find y_n when $y = x^4 \cos 3x$.

b) If $y = \log(x + \sqrt{x^2 + 1})$, then show that

$(x^2 + 1)y_{n+2} - (2n+1)xy_{n+1} + n^2 y_n = 0.$ 4+6

9. a) State and prove Leibnitz's theorem on successive differentiation.

b) Are the conditions of Rolle's theorem satisfied in the following function

$f(x) = 1 - x^{\frac{2}{3}}$ in $[-1, 1]$. 6+4

10. a) Prove that $x \le \sin^{-1} x \le \frac{x}{\sqrt{1-x^2}}$, if $0 \le x \le 1$.

b) Expand the function $\sin^3 x$ in power of x in a finite form with Lagrange's form of remainder. 5+5

11. a) Show that $4^x - 8x\log_e 2$ is a minimum when $x = 1$.

b) Find the value of p, q, r so that

$\frac{(p + q\cos x) - r\sin x}{x^3} \to 1$ as $x \to 0$. 5+5

[Turn over

7. a) Find the eigenvalues and the corresponding eigenvectors of the following matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

b) If $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the eigenvalues of a non-singular matrix A of order n, find the eigenvalues of A^{-1}. 6+4

b) Find the range of values of x for which

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7$$

is concave upwards or downwards. Find also the points of inflexion, if any. 5+5

5. a) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$ then show that

$$\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}} = (2a)^{-\frac{2}{3}}.$$

b) Show that the points of inflexion of the curve

$$y^2 = (x-a)^2(x-b)$$

lie on the line $3x + a = 4b$. 5+5

6. a) If the straight line $lx + my = n$ touches the curve

$$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^p = 1$$

then prove that

$$(al)^{\frac{p}{p-1}} + (bm)^{\frac{p}{p-1}} = n^{\frac{p}{p-1}}.$$

b) Find the radius of curvature of $y = xe^{-x}$ at a point where y is maximum. 5+5

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