B.E. PRODUCTION ENGINEERING Examination 2019
(1st year, 2 nd Semester (Old))
MATHEMATICS III

Time: Three Hours **Full Marks: 100**

Use a separate Answer-Script for each part.

Part – I

(Answer question no. 6 and any three from rest)
(Symbols/Notations have their usual meanings)

1 a) If $\bar{a}$ is a constant vector, prove that

$$\bar{\nabla} \text{X} \left(\frac{\bar{a}\, X\, \bar{r}}{r}\right) = \frac{\bar{a}}{r} + \frac{\bar{a}\,.\,\bar{r}}{r^2}\ \bar{a}.$$ 6

b) Show that the vector $r^n\,\vec{r}$ is an irrotational for any value of n but is solenoidal if $n + 3 = 0$. 6

c) Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2$ at $(2, -1, 2)$ in the direction $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. 4

2 a) Show that under rotation of rectangular axes the origin remaining the same, the vector d ifferential operator $\vec{\nabla}$ remains invariant. 6

b) Evaluate $\int_2^3 (\vec{r} \times \frac{d^2\vec{r}}{dt^2})\, dt$, where $\vec{r}(t) = t^3\vec{\imath} + t^2\vec{\jmath} + t\vec{k}$. 6

c) Show that $\vec{\nabla}.(\vec{F} \times \vec{G}) = \vec{G}\,.\,\vec{\nabla} \times \vec{F} - \vec{F}\,.\,\vec{\nabla} \times \vec{G}$
Hence show that, if the vectors $\vec{F}$ and $\vec{G}$ are irrotational , show that $\vec{F} \times \vec{G}$ is solenoidal. 4

3 a) Evaluate $\iint\limits_S (\bar{\nabla} \times \bar{F})\,.\,\bar{n}\, ds$, where $\vec{F} = y\vec{\imath} + (x - 2xz)\vec{\jmath} - xy\,\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = z^2$ above xy –plane. 8

b) Verify Stokes' theorem for $\bar{F} = (y - z + 2)\,\hat{\imath} + (yz + 4)\hat{\jmath} - xz\,\hat{k}$ over the surface of the cube $x = y = z = 0$ and $x = y = z = 2$ above xy-plane. 8

[Turn over

4 a) If $F(s)$ is Fourier transform of $f(x)$, then show that $F[f(ax)] = \frac{1}{a} F(s/a)$. 8

b) Find Fourier transform of

$$f(x) = 1 - x^2 \quad if \ |x| < 1$$
$$= 0 \quad if \ |x| \geq 1$$

8

5. a) Using convolution theorem evaluate $Z^{-1}[\frac{z^2}{(z-1)(z-3)}]$ 8

b) Solve $4U_k - U_{k+2} = 0, \ if \ u_0 = 0, u_1 = 2$ 8

6. Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint x\,dy - y\,dx$. 2

Part – II

Answer any 5 questions.

7. Expand the function $f(x) = e^{-x}$ as a Fourier series in the interval $0 < x < 2\pi$. 10

8. Obtain a Fourier series for the function

$$f(x) = 1 + \frac{2x}{\pi} \qquad -\pi \leq x \leq 0,$$
$$= 1 - \frac{2x}{\pi} \qquad 0 \leq x \leq \pi.$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. 8+2

9. Find the half range sine and cosine series for the function given below

$$f(x) = x, \qquad 0 \leq x \leq \frac{\pi}{2},$$
$$= \pi - x, \qquad \frac{\pi}{2} \leq x \leq \pi.$$

5+5

10. Solve completely the equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, representing the vibrations of a string of length l, fixed at both ends, given that $y(0,t) = 0$; $y(l,t) = 0$; $y(x,0) = f(x)$ and $\frac{\partial y}{\partial t}(x,0) = 0, 0 < x < l$. 10

11. A tightly stretched string with fixed end points $x = 0$ and $x = \boldsymbol{a}$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $\lambda x(a - x)$, find the displacement of the string at any distance x from one end at any time t. 10

12. Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 y}{\partial x^2}$ where the boundary conditions are $u(0,t) = 0;\ u(l,t) = 0\ (\ t > 0);$ and the initial condition $u(x,0) = x\ , l$ being the length of the bar.

10

13. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0\ ,\ where\ 0 \le x \le a, 0 \le y \le b$, given that

$$u(0,y) = u(a,y) = u(x,0) = 0\ and\ u(x,b) = sin\frac{n\pi x}{a}.$$ 10