Bachelor of Production Engineering Examination, 2019

(1st Year, 1st Semester)

MATHEMATICS II

Time : Three hours Full Marks : 100

(Symbols/ Notations have their usual meanings)

*Answer **any five** questions*

1.Solve the following differential equations:

(a) $(x^2 + y^2 + 2x)dx + 2ydy = 0$

(b) $y\,dx - xdy + \log x\,dx = 0$

(c) $(1 + xy)y\,dx + (1 - xy)x\,dy = 0$

(d) $(3xy - 2ay^2)dx + (x^2 - 2axy)dy = 0$

5+5+5+5

2.(a) Solve the following differential equations:

(i) $(D^2 + 5D + 6)y = e^{-2x}\sin 2x$

(ii) $(D^3 + D^2 - D - 1)y = \cos 2x$

(iii) $(D^2 + D + 1)y = (1 - e^x)^2$

(b) Solve $\frac{d^2y}{dy^2} - 2\frac{dy}{dx} = e^x \sin x$, using method of variation of parameters.

(5+5+5)+5

3.(a) Find the Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 2t)$

(b) Find $L^{-1}\left[\frac{s+1}{s^2 + 6s + 25}\right]$

(c) Use convolution theorem to evaluate

$$L^{-1}\left[\frac{s^2}{(s^2+4)(s^2+9)}\right]$$

(d) Using Laplace Transform, solve the differential equation

$\frac{d^2y}{dy^2} + 6\frac{dy}{dx} + 9y = 1$, given that $y(0) = 0$, $y'(0) = 1$.

5+5+5+5

4.(a) Find the series solution of the equation

$$xy'' + y' - y = 0$$

in powers of x, about $x = 0$.

(b) Find series solution about $x = 0$ of the differential equation

$$y'' + xy' + x^2y = 0$$

10+10

5 (a) Prove that

$$\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = 0, \; if \quad m \neq n$$
$$= \frac{2}{2n+1}, \; if \quad m = n$$

(b) Prove that (i) $nP_n = (2n-1)\,xP_{n-1} - (n-1)P_{n-2}, \quad n \geq 2$

(ii) $nP_n = x\,P_n' - P_{n-1}'$

(5+5)+(5+5)

6. (a) Show that the function $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$ $(z \neq 0)$, $f(0) = 0$, satisfies Cauchy-Riemann equations at the origin but $f'(0)$ does not exist.

(b) If $u = e^x(x\cos y - y\sin y)$, find v such that *f(z)* = $u+iv$ is analytic.

(c) If *f(z)* is an analytic function of *z*, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

8+6+6

7. (a) Using Cauchy's integral formula, evaluate the integral $\int_{|z|=3} \dfrac{e^{2z}}{(z-1)(z-2)}\,dz$.

(b) State Cauchy's Residue theorem. Use this theorem to evaluate $\int_{|z-i|=2} \dfrac{1}{(z^2+4)^2}\,dz$.

(c) Expand $f(z) = \dfrac{(z-2)(z+2)}{(z+1)(z+4)}$ in Taylor's/Laurent's series valid in the region

(i) $|z| < 1$, (ii) $1 < |z| < 4$ (iii) $|z| > 4$.

6+8+6