B.Mechanical(Evening) Examination, 2019(old)

(1ST YR, 1ST SEM)

MATHEMATICS

PAPER - VM

Full Marks : 100 **Time: Three hours**

Answer any ten questions **$10 \times 10 = 100$**

1. What do you mean by subspace of a vector space? State a necessary and sufficient condition for a non empty subset W of a vector space V(F) be a subspace of V. Show that intersection of two subspaces is also a subspace.

2. Define vector space.
Let
$$S = \{(x, y, z)/x + y + 2z = 0\}.$$
Show that S is a subspace.
Show also that
$$S = \{(x, y, z)/x^2 + y^2 = z^2\}.$$
is not a subspace.

3. Define linear dependent and independent vectors. Check whether the vectors are dependent or independent:
$$(-1, 2, 1), \quad (3, 0, -1), \quad (-5, 4, 3).$$
Show also, the union of two subspaces is not a subspace.

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4. Define linear span L(S) of a subset S of a vector space.
Let $S = (\alpha, \beta)$ and $T = (\alpha, \beta, \alpha + \beta)$ Then show that $L(S) = L(T)$.
Find a basis containing the vectors $(1, 1, 0)$ and $(1, 1, 1)$.

5. What do you mean by inner product space? Define norm of a vector. Prove that

$$|(\alpha, \beta)| \leq ||\alpha||||\beta||.$$

6. (a) Define linear mapping. Show that $T : R^3 \to R^3$ defined by

$$T(x, y, z) = (x + 1, y + 1, z + 1)$$

is a not linear mapping.

(b) Show that $T : R^2 \to R^3$ defined by

$$T(x, y, z) = (2x + y + z, x + 2y + z),$$

is a linear mapping. Find Ker T and dim Ker T.

7. (a) Show that the necessary and sufficient condition for a vector function $\vec{F}(t)$ to have constant magnitude is

$$\vec{F}(t) \cdot \frac{d\vec{F}(t)}{dt} = 0$$

(b) Find the directional derivative of a scalar point function $f(x, y, z) = xy + yz + zx$ in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$ at (1,2,0).

8. Prove that:

$$(i)\ curl\ grad\ \phi = 0 \quad (ii)\ div\ curl\ \vec{F} = 0$$

9. State Stoke's theorem. Verify Stoke's theorem where

$$\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$$

and the surface S is the part of the sphere

$$x^2 + y^2 + z^2 = a^2$$

10. (a) Find the angle between two surfaces

$$xy^2z = 3x + z^2 \ \ and \ \ 3x^2 - y^2 + 2z = 1 \ \ at \ (1, -2, 1).$$

(b) If $\vec{r} \times d\vec{r} = 0$, prove that $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ is a constant vector.

11. (a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2+y^2)\,\vec{i} - 2xy\,\vec{j}$ and the curve C is the rectangle in the xy plane bounded by $y = 0,\ x = a,\ y = b,\ x = 0$.

(b) If $\vec{F} = (2x^2 - 3z)\,\vec{i} - 2xy\,\vec{j} - 4x\,\vec{k}$, then evaluate

$$\int_V \vec{\nabla} \times \vec{F}\, dV$$

where V is a closed region bounded by the planes $x = 0,\ y = 0,\ z = 0,$ and $2x + 2y + z = 4$.

12. State Gauss Divergence theorem. Verify Gauss Divergence theorem for $\vec{F} = 4x\,\vec{i} - 2y^2\,\vec{j} + z^2\,\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4$, z=0 and z=3.