

B.Mechanical(Evening). Examination, 2019

(1ST YR, 1ST SEM)

MATHEMATICS

PAPER - III

Full Marks : 100

Time: Three hours

Part - I

Answer any four questions

12.5 × 4 = 50

Symbols / Notations have their usual meanings.

1.(a) Find the Z-Transformation of the following functions:

$$(i) f(n) = n \quad (ii) f(n) = a^n$$

(b) Solve the equation using Z-Transformation

$$f(n + 1) + 2f(n) = n, \text{ given : } f(0) = 1.$$

2. Find the Laplace Transformations of the following functions:

$$(i) f(t) = t^3$$

$$(ii) f(t) = \sin \sqrt{t}.$$

$$(iii) f(t) = \frac{t}{T}, \quad 0 < t < T \\ = 1, \quad t > T$$

3. (a). Find the Fourier Transformations of the following function

$$e^{-|t|}$$

[Turn over

(b) Solve the equation using Laplace Transformation:

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = 0 \quad \text{given : } x(0) = x'(0) = 1$$

4. Find the Fourier series of the function

$$f(x) = x \sin x, \quad \text{when } -\pi < x < \pi$$

Deduce also

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots$$

5.(a) Find $L[F''(t)]$, where L stands for Laplace Transformation.

(b) Find

$$L^{-1}\left(\frac{1}{\sqrt{2s+3}}\right).$$

Part-II

Answer any two questions:

15 × 2 = 30

6. Solve the equations:

$$(i) \frac{dy}{dx} = \sin(x+y) \quad (ii) xdy - ydx = \sqrt{x^2 + y^2}$$

7. (a). Find general solution and singular solution of the following equation.

$$p = \ln(px - y), \quad \text{where } p = \frac{dy}{dx}$$

(b). Find the general solution of the following equation.

$$(D^2 + D - 6)y = x^2, \quad \text{where } D = \frac{d}{dx}$$

8. Find the series solution near the ordinary point $x=1$ of the equation

$$\frac{d^2y}{dx^2} + (x-1)^2 \frac{dy}{dx} - 4(x-1)y = 0$$

Part-III

Answer the following questions:

10 × 2 = 20

10. Solve the following equation.

$$(i) (y + z)p + (z + x)q = x + y, \quad (ii) z^2 - pz + qz + (x + y)^2 = 0$$

$$\left[\text{where } p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y} \right]$$

11. Solve the Heat flow equation using the method of separation of variables.