# BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING)

## 5<sup>TH</sup> YEAR 1<sup>ST</sup> SEMESTER EXAMINATION- 2019

# **Subject: DIGITAL CONTROL TECHNIQUES**

Time: Three Hours Full Marks: 100

## Answer Any Five questions (5×20)

Question
No.
Q1 (a) What is an Impulse Sampler? Show how it can also be referred to as an Impulse Modulator.
(b) Given e(k)=1 for all k, find E(z). Justify the result by using Initial Value Theorem and Final Value Theorem.
(c) Obtain the transfer function C(z)/R(z) for the closed loop configuration shown in Figure Q1(c).

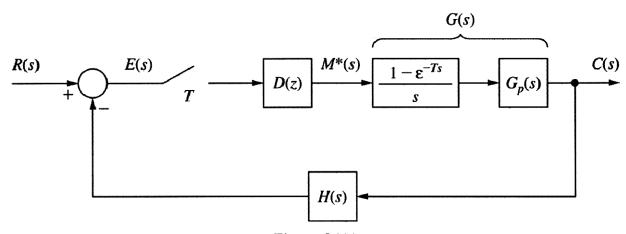


Figure Q1(c)

- Q2 (a) Show that for a Type-0 continuous-time system the sample and hold operation does not affect the static error constants.
  - (b) Find the expression for e(k) for

$$E(z) = \frac{z}{(z-1)(z-2)}$$

(c) Given the system shown in Figure Q2(c), with input e(t) being a unit step function, determine the output function C(z).

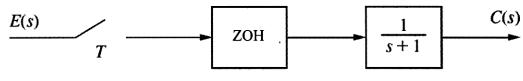


Figure Q2(c)

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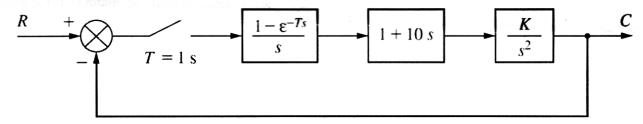
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- Q3 (a) State and explain Jury's Stability Test for a closed-loop discrete-time system.
  - (b) The closed loop characteristic equation of a discrete-time system is given as

$$Q(z) = z^3 - 1.8z^2 + 1.05z + 0.2 = 0$$

Determine whether the system is stable or not.

- Q4 (a) Show how the left half of the s-plane will be mapped into the z-plane.
  - (b) Consider the system shown in Figure Q4(b). Draw the root locus and determine the range of K for which the system remains stable.



### Figure Q4(b)

Q5 (a) Consider the discrete-time system defined by

$$\frac{Y(z)}{U(z)} = \frac{b_0 z^n + b_1 z^{n-1} + \dots + b_n}{z^n + a_1 z^{n-1} + \dots + a_n}$$

Show that a state-space representation of this system may be given by

$$\begin{bmatrix} x_{1}(k+1) \\ x_{2}(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_{n}(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_{n} & -a_{n-1} & -a_{n-2} & \cdots & -a_{2} & -a_{1} \end{bmatrix} \begin{bmatrix} x_{1}(k) \\ x_{2}(k) \\ \vdots \\ x_{n-1}(k) \\ x_{n}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(k)$$
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$$y(k) = [(b_n - a_n b_0) \quad (b_{n-1} - a_{n-1} b_0) \quad \cdots \quad (b_1 - a_1 b_0)] \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + b_0 u(k)$$

(b) Obtain the state-variable model of the system described by the difference equation

$$y(k + 2) = u(k) + 1.7y(k + 1) - 0.72y(k)$$

where, u(k) is the input and y(k) is the output of the system.

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Q6 (a) The state-space representation of an *n*-th order linear time-invariant discrete-time system is given as

$$x(k+1) = Gx(k) + Hu(k),$$
  

$$y = Cx(k) + Du(k)$$
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Obtain the *Pulse Transfer Function* of the system. State the necessary assumptions.

- (b) Show that the Pulse Transfer Function is invariant under the similarity transformation  $x(k) = P\hat{x}(k)$ .
- (c) Obtain the transfer function for the system described by the state equations

$$x(k+1) = \begin{bmatrix} 1.35 & 0.55 \\ -0.45 & 0.35 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(k)$$

- Q7 (a) Define, with the help of a block diagram, the State and Output equations in respect of a liner discrete-time invariant system.
  - (b) Consider the continuous-time system given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+a}$$

- (i) Obtain the continuous-time state space representation of the system.
- (ii) Discretize the state and output equations to obtain the discrete-time state 2+4+8 space representation of the system.
- (iii) Determine the pulse transfer function of the system and show that it is identical to the transfer function of the system obtained by taking z-transform of G(s) preceded by a sampler and a ZOH.
- O8 Write short notes on any two from the following:
  - (a) Zero Order Hold circuit as a Data Hold Circuit.
     (b) Significance of Bilinear Transformation in respect of discrete-time systems.
  - (c) Various types of analog and digital signals associated with a discrete-time control system.
  - (d) Non-uniqueness of discrete-time state-space representation.