Ref. No.: EX/EE/5/T/522A/2019

BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING)
5TH YEAR 2ND SEMESTER EXAMINATION- 2019
Subject: ADVANCED CONTROL THEORY

Time: Three Hours **Full Marks: 100**

Answer ***Any Five*** questions (5×20)

Question No.			Marks
Q1	(a)	Briefly discuss about the common non-linearities found in physical systems.	10
	(b)	Define Phase Plane and Phase Trajectory?	4
	(c)	Draw the phase portrait of a spring-mass system given by $\dot{x} + x = 0$, with $x(0) = x_0$.	6
Q2	(a)	What is Limit Cycle? With the help of proper phase-portraits define Stable, Unstable and Semi-Stable Limit Cycles.	3+9
	(b)	Consider the following non-linear system: $\dot{x}_1 = x_2 - x_1({x_1}^2 + {x_2}^2 - 1) \qquad \dot{x}_2 = -x_1 - x_2({x_1}^2 + {x_2}^2 - 1)$ Show that the system has a stable limit cycle.	8
Q3	(a)	Discuss the concept of Describing function for non-linear systems?	4
	(b)	Obtain the Describing Function for Saturation type non-linearity and draw the normalized plot of the same.	10
	(c)	Show how Describing function can be used to predict the stability of limit cycles.	6
Q4	(a)	Consider a linear autonomous system described by the state equation $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ Show that the system is asymptotically stable in-the-large at the origin *if and only if* given any symmetric, positive definite matrix $\boldsymbol{Q}$, there exists a symmetric positive definite matrix $\boldsymbol{P}$ which is the unique solution of $\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{Q}$	8
	(b)	Show that the system given by $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ with $\boldsymbol{A} = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix}$ is asymptotically stable in-the-large at the origin.	6
	(c)	Consider a non-linear system given by the following equations: $\dot{x}_1 = -x_1 + 2{x_1}^2 x_2 \qquad \dot{x}_2 = -x_2$ Show the region of stability for the system.	6

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Q5 (a) With the help of an example describe the operation of On-Off Controller. Discuss about the advantages and disadvantages of such control scheme. 4+4

(b) Briefly describe the steps involved in linearization of a non-linear model involving non-linear function of a single variable. 4

(c) Consider a single tank liquid-level system where the outflow passes through a valve. Assume that the valve discharge rate is related to the square root of liquid level, $q = C_v\sqrt{h}$, where C_v depends on the fixed opening of the valve. Derive an approximate dynamic model for this process by linearization. 8

Q6 (a) Distinguish between structured and unstructured uncertainty. With the help of block diagrams explain Additive and Multiplicative unstructured uncertainties. 4+10

(b) Consider a family of plant transfer functions given by

$$\frac{1}{s^2 + as + 1}, \qquad 0.4 \le a \le 0.8$$ 6

Represent the original plant as a feedback uncertainty around a nominal plant.

Q7 (a) Prove that the distance from -1 to the Nyquist plot of L equals $1/\|S\|_\infty$, where L is the loop transfer function and S is the sensitivity function for any system. 8

(b) Briefly discuss the concept of Small Gain Theorem. Show how it can be extended to robust control problem for plants with (i) additive and (ii) multiplicative uncertainties. 4+4+4

Q8 (a) State the Quadratic Optimal Regulator Problem. 6

(b) Consider the system

$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t)$, with $\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Assuming the control signal to be

$$u(t) = -\boldsymbol{K}\boldsymbol{x}(t)$$ 10

determine the optimal feedback gain matrix $\boldsymbol{K}$ such that the following performance index is minimized:

$$\int_0^\infty (\boldsymbol{x}^T\boldsymbol{x} + u^2)dt$$

(c) Given a system in state equation form $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t)$, with

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ 4

Show that the system cannot be stabilized by state-feedback control $u(t) = -Kx(t)$.