Ref No: Ex/EE/5/ T/ 421 /2019

BACHELOR OF ENGINEERING (ELECTRICAL ENGINEERING) FOURTH YEAR, SECOND SEMESTER EXAMINATION, 2019

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Full Marks 100

Time: Three hours **(50 marks for each part)**

Use a separate Answer-Script for each part

Question No. **PART- I** **Marks**

Answer any **THREE** questions

Two marks reserved for neatness and well-organized answers

1. (a) What is meant by the relative frequency of occurrence of a random event? How is it related with the 'Probability' of the random event? Explain the terms joint probability and conditional probability, and establish a relation between them. **9**

(b) A visual inspection of a location on wafers from a semiconductor manufacturing process resulted in the following table: **7**

Number of Contamination Particles	Proportion of Wafers
0	0.40
1	0.20
2	0.15
3	0.10
4	0.05
5 or more	0.10

If one wafer is selected randomly from this process and the location is inspected, what is the probability that

(i) it contains no particles in the inspected location?
(ii) it contains less than five particles in the inspected location?
(iii) it contains three or more particles in the inspected location?

2. (a) Derive an expression for obtaining the expectation of a random variable from knowledge of its probability mass function. Also derive the same for calculating the expectation of a continuous random variable from knowledge of its probability density function. **8**

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Questions No.	PART- I	Marks

(b) **Suppose the cumulative distribution function of a random variable X is**

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.25x + 0.5 & -2 \leq x < 1 \\ 0.5x + 0.25 & 1 \leq x < 1.5 \\ 1 & 1.5 \leq x \end{cases}$$ **8**

Determine the following.

(i) Expression for the probability density function of X.
(ii) Expectation of X.
(iii) P $(-1 < X < 1.25)$.
(iv) P$(X>0.5)$.

3. (a) **What is *'moment generating function'* of a random variable? Is it related to the any transform of the probability density function? How can it be utilized to obtain the moments of different orders of the random variable?** **6**

(b) **The line width for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 µm and a standard deviation of 0.05 µm.** **10**

(i) What is the probability that a line width exceeds 0.62 µm?

(ii) What is the probability that a line width is between 0.47 and 0.63 µm?

(iii) What is the value below which line width of 90% samples are expected to lie?

Use the attached table for Normal distribution. Use linear interpolation wherever necessary.

4. (a) **What is the expression for the probability density function of an exponentially distributed random variable Y ? Derive expressions for the expectation and the standard deviation of Y in terms of the parameter of the distribution.** **4+4**

Derive and explain the *'Lack-of-Memory'* property of exponential distribution.

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No. of Questions | PART- I | Marks

(b) **Write down the expression for the probability mass function of binomial random variable. From this expression, obtain the expressions for the cumulative distribution function, the expectation and the variance of binomial random variable, in terms of the distribution function parameters.** 8

5. **Answer *any two* of the following.**

(a) **List the properties of probability density function.**

(b) **What is meant by statistical independence of two random variables? If $U=X+Y$, where X and Y are statistically independent random variables, show that the pdf of U is the convolution of the pdfs of X and Y.**

(c) **Discuss the uniform probability distribution and its application in modeling ADC quantization noise.** 8+8

(d) **Consider the probability density function**

$$f(x)=\frac{1}{b\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-a}{b}\right)^2\right);\ -\infty\leq x\leq+\infty\ ,$$

where $-\infty< a <+\infty$ & $0 < b <+\infty$.

Determine the expectation and the standard deviation of X, in terms of a and b.

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Table Standard normal distribution **F(z) for z= 0.00– 2.99**

Z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9820	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

Note:

F(z) **is the cumulative distribution function of the standard normal random variable.**

That is,

$$F(z) = P(Z \le z)$$

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BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING (EVENING) EXAMINATION, 2019

(4th Year, 2nd Semester)

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHOD

Time: Three Hours Full Marks: 100

(50 marks for each part)

Use a separate Answer-script for each Part

PART-II

Answer any *three questions*

(*Two marks are reserve for neatness and well organized answers*)

1. a) A and B are the least square estimators of α and β in a linear regression model and the random errors are independent normal random variables having mean '0' and variance σ^2. Establish that A and B are unbiased estimators of α and β. 8

b) The following data relate the moisture of a wet mix of a certain product (x) to the density of the finished products (y). Represent a linear equation to fit the data. 8

x	5	6	7	10	12	15	18	20
y	7.4	9.3	10.6	15.4	18.1	22.2	24.1	24.8

2. a) Establish the relationship between sample variance and population variance. State and discuss central limit theorem. 8

b) An insurance company has 25,000 automobile policy holder. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8.3 million. 8

3. a) Suppose $X_1, \ldots, X_n$ are independent Poisson random variables each having a mean of λ. Determine the maximum likelihood estimator of λ. 8

b) A signal having value μ is transmitted from location A and the value received at location B is normally distributed with mean μ and variance 4. If the successive values received are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, and 10.5, find 95% confidence interval for μ. 8

[Turn over

4. a) Determine confidence interval for a normal mean when the variance is unknown. 8

b) A sample of 10 grains of metallic sand taken from a large sand pile have respective lengths (in mm)

2.2, 3.4, 1.6, 0.8, 2.7, 3.3, 1.6, 2.8, 2.5, 1.9

Estimate the percentage of sand grains in the entire pile whose length is between 2 and 3 mm. 8

5. Write short notes on **any two** of the following: 2x8=16

a) The Chi-Square Distribution

b) Statistical inferences concerning α and β.

c) The F – distribution