d) An urn contains n tickets, numbered 1 to n and m tickets are drawn at a time. Find the mathematical expectation of the sum of numbers on the tickets drawn. 5

7. a) If r be the distance of $P(x, y, z)$ from the origin and $\vec{r}$ be the position vector of P relative to the origin, then find $\vec{\nabla}^2\left(\frac{1}{r}\right)$. 4

b) Two unbiased dice are thrown. Find the conditional probability that two fives occur if it is divisible by 5. 5

c) Evaluate $\iint_S (y^2z^2\hat{i} + z^2x^2\hat{j} + z^2y^2\hat{k}).\hat{n}\, dS$ where S is the part of the sphere $x^2 + y^2 + z^2 = 1$, above the xy-plane and boundary of this plane. 8

d) Show that $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{0}$. 3

Ex/EE/5/MATH/T/113/2019(Old)

Bachelor of Engineering (Electrical Engineering) Examination, 2019

(1st year, 1st Semester, Old)

Mathematics - IVF

Time : Three hours | Full Marks : 100

(Symbols and notations have their usual meanings)

Answer ***any five*** questions.

1. a) Evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 6

b) Show that the vector $\vec{F} = (4xy - z^3)\hat{i} + 2x^2\hat{j} - 3xz^2\hat{k}$ is irrotational. also show that $\vec{F}$ can be expressed as the gradient of some scalar point funcion φ. 7

c) Find the directional derivatives of the function $f = xy^2z + 4xz^2$ at the point (–2, 1, 2) in the direction $2\hat{i} + \hat{j} - 2\hat{k}$. 7

2. a) Show that

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \vec{\nabla}^2 - \vec{\nabla}^2\vec{F}.$$ 7

b) Verify Stoke's theorem for the vector function $\vec{F} = (x^2 - y^2)\hat{i} + 2x\hat{j}$ around the rectangle bounded by straight lines $x = 0, x = a, y = 0, y = b$. 8

[Turn over

c) Find the unit vector perpendicular to each of

$\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 6\hat{j} - 2\hat{k}$. 5

3. a) Let C be the arc of the circle $|z| = 4$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Show that

$$\left| \int_C \frac{z^2}{(z-2)(z+3)} dz \right| \le \frac{\pi}{3}.$$ 7

b) Using contour integration evaluate

$$\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}.$$ 8

c) Verify Cauchy-Riemann equations for the function

$f(z) = z^2$. 5

4. a) Show that

$$f(z) = \begin{cases} \frac{z\,\mathrm{Re}\{z\}}{|z|} & \text{if} \quad z \neq 0 \\ 0 & \text{if} \quad z \neq 0 \end{cases}$$

is continuous at $z = 0$ but not difrerentiable at $z = 0$. 8

b) Evaluate $\int_C \frac{e^z}{z^2 + 4} dz$ where C is positively oriented circle 7

c) Expand $\frac{\text{[redacted]}}{(z-1)(z-2)}$ as a power series in z in the region $1 < |z| < 2$. 5

5. a) Find the mean and variance of Poisson distribution. 8

b) Find the probability that in a game of bridge, a hand of 13 card will contain (i) all the aces (ii) at least one ace. 7

c) For any events A and B, show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$ 5

6. a) Define Exhaustive events, Null events. 4

b) A box contains 3 white, 5 red and 4 black balls. One ball is drawn at random and is replaced before the next draw. Find the expectation and variance of the number of white balls if the experiment be repeated 80 times in succession. 6

c) Find the value of the constant K such that

$$f(x) = \begin{cases} Kx(1-x), & 0 < x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function and compute

$$P\left(X > \frac{1}{2}\right).$$ 5