b) Find an integrating factor and hence

solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ 8+8

9. Find the angle between the lines

$$\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4}$$

and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

OR

If $\vec{a} = 5t^2\vec{i} + t\vec{j} - t^2\vec{k}$ and $\vec{b} = \sin t\,\vec{i} - \cos t\,\vec{j}$ then find

$\frac{d}{dt}(\vec{a}\cdot\vec{b})$ and $\frac{d}{dt}(\vec{a}\times\vec{b})$ 4

Ex/EE/5/Math/T/113/2019

Bachelor of Engineering in Electrical Engineering Examination, 2019

(1st Year, 1st Semester)

Mathematics - IIF

Time : Three hours Full Marks : 100

Answer question no. ***9*** and any ***six*** from the rest.

1. a) Find the inverse of the matrix

$$A = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 6

b) Find the rank of the matrices

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 4 & 5 & 2 & 3 \end{pmatrix}.$$

Use this to conclude that the system of equations

$2x + 3y + z = 4$

$4x + 5y + 2z = 3$

is consistent.

Why does the system have infinitely many solutions ?

(3+2)+2+3

2. a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{pmatrix} 0 & 6 \\ -1 & 5 \end{pmatrix}$.

Also verify Caley Hamilton theorem for A and hence find the inverse of A. 6+3+3

[Turn over

b) Find the direction ratios and direction cosines of the line joining the points (1, 2, 3) and (2, –3, 4) 2+2

3. a) Find the equation of the plane passing through the point (3, –3, 1) and perpendicular to the line joining the points (3, 4, –1) and (2, –1, 5) 5

b) A line is given by $x-2y+z=0,\ \ x+2y-2z=0$. Find its equation in symmetric form. 5

c) Show that the line $\frac{x-2}{-1}=\frac{y-1}{6}=\frac{z-3}{-6}$ lies in the plane that passes through the points A(1, –2, 3), B(1, 1, 1) and C(0, 1, –1). 6

4. a) If $\vec{f}=(3x^2+6y)\vec{i}-14yz\vec{j}+20xz^2\vec{k}$, evaluate $\int_C \vec{f}\cdot d\vec{r}$ where C is given by $x=t, y=t^2$ and $z=t^3$ and t varies from 0 to 1. 5

b) Evaluate $\int (x^2+xy)dx+(x^2+y^2)dy$ where C is the square formed by the lines $y=\pm 1, x=\pm 1$ traversed anticlockwise. 5

c) Find the work done in moving a particle in the force field $\vec{f}=3x^2\vec{i}+(2xz-y)\vec{j}+z\vec{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). 6

5. a) Evaluate $\iint_S \vec{f}\cdot\vec{n}\,ds$ where $\vec{f}=18z\vec{i}-12\vec{j}+5y\vec{k}$ and S is the surface $2x+3y+6z=12$ surface in the 1st octant. 9

b) Evaluate $\oint_C [(3x-y)dx+(2x+y)dy]$ where C is the curve $x^2+y^2=a^2$. 7

6. Verify Green's theorem in the plane for $\oint_C [(3x^2-8y^2)dx+(4y-6xy)dy]$ where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1. 16

7. a) Using Gauss's Divergence theorem evaluate $\iint_S \vec{f}\cdot\vec{n}ds$ where $\vec{f}=4xz\vec{i}-y^2\vec{j}+yz\vec{k}$ and S is the surface of the cube $x=0, x=1, y=0, y=1, z=0, z=1$. 8

b) Evaluate $\oint_C \vec{F}\cdot d\vec{r}$ by Stoke's theorem where $\vec{F}=y^2\vec{i}+x^2\vec{j}-(x+z)\vec{k}$ and C is the boundary of the triangle (0, 0, 0), (1, 0, 0) and (1, 1, 0) transversed anticlockwise. 8

8. a) Solve $(D^3-6D^2+11D-6)y=e^{-2x}+e^{-3x}$

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