Ref. No.: EX/CE/MATH/5/T/101/2019(OLD)

BACHELOR OF CIVIL ENGINEERING (EVENING) EXAMINATION – 2019(OLD)

(1^{ST} YEAR 1^{ST} SEMESTER)

MATHEMATICS – I

FULL MARKS : 100 | TIME : 3 HOURS

Answer any *six* questions.
Four marks are reserved for neatness.
(Notations have their usual meanings)

1. Solve the following differential equations :

(a) $(1 - xy)ydx - (1 + xy)xdy = 0$

(b) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

(c) $(x^2 - 3y^2)dx + 2xydy = 0.$

(d) $(xy^2 - e^{\frac{1}{x^3}})dx - x^2ydy = 0$ 4X4

2. Solve the following differential equations :

(a) $\left(x + y\cos\frac{y}{x}\right)dx = x\cos\frac{y}{x}dy$

(b) $x\cos x\,\frac{dy}{dx} + y\,(x\sin x + \cos x) = 1$

(c) $(D^2 - 7D + 6)y(x) = 2e^{3x}$, when $y(0) = 1$ and $(\frac{dy}{dx})_{x=0} = 0$ 5+5+6

3. Solve the following differential equations:

(a) $(D^4 - 2D^3 + D^2)y(x) = x^3$

(b) $(D^2 - 2D + 4)y(x) = e^x\cos^2 x.$ 8+8

4. (a) Prove that

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi}\cos(n\theta - x\sin\theta)d\theta$$

(b) Prove that

$$J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$$ 10+6

5. (a) Show that for a Legendre function $P_n(x)$

$$(1 - 2xh + h^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} h^n P_n(x)$$

(b) Prove that $x\,P_n'(x) = P_{n-1}'(x) + nP_n(x).$ 10+6

6. (a) Find the Fourier Series for $f(x) = x - x^2$ in $-\pi < x < \pi$. Hence deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots\dots\dots\dots$$

(b) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$, in terms of Legendre polynomials. 10 + 6

7. (a) Determine the series solution of the equation about x = 0

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0.$$

(b) Prove that $P_n(z) = \frac{1}{2^n n!}\frac{d^n}{dz^n}(z^2 - 1)^n$. 10+6

8. Solve :

(a) L [sin2t cos3t]

(b) $L^{-1}\left[\frac{1}{s(s^2+9)}\right]$

(c) $y''(t) - 2y'(t) - 8y(t) = 0$ subject to the initial conditions y(0) = 3 and $y'(0) = 6$.

5 + 5 + 6