INVESTIGATIONS ON SOME IMPORTANT PROPAGATION PARAMETERS OF OPTICAL FIBERS CONCERNED WITH OPTICAL TECHNOLOGY AND ASSOCIATED DEVICES IN THE FRAMEWORK OF SIMPLE THEORETICAL MODELING

THESIS SUBMITTED BY

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1. "A simple but accurate method for prediction of splice loss in single-mode dispersion shifted trapezoidal as well as dispersion flattened graded and step W fibers", **Subhalaxmi Chakraborty**, Angshuman Majumdar and Sankar Gangopadhyay, **Optik (Elsevier)**, 124 (2013), pp. 6752-6756.

2. "A novel and accurate method for analysis of single-mode dispersion-shifted and dispersion-flattened fiber directional coupler", **Subhalaxmi Chakraborty**, Shubhendu Maiti, Chintan Kumar Mandal and Sankar Gangopadhyay, **Optik (Elsevier)**, 157 (2018), pp. 808-816.

3. "Prediction of fundamental modal field for graded index fiber in presence of Kerr nonlinearity", **Subhalaxmi Chakraborty**, Chintan kumar Mandal and Sankar Gangopadhyay, **J. Opt. Commun. (Germany)**, doi: 10.1515/JOC-2017-0126.

4. "Prediction of first higher order modal field for graded index fiber in presence of Kerr nonlinearity", **Subhalaxmi Chakraborty**, Chintan kumar Mandal and Sankar Gangopadhyay, **J. Opt. Commun. (Germany)**, doi: 10.1515/JOC-2017-0206.

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CERTIFICATE FROM THE SUPERVISORS

This is to certify that the thesis entitled "**INVESTIGATIONS ON SOME IMPORTANT PROPAGATION PARAMETERS OF OPTICAL FIBERS CONCERNED WITH OPTICAL TECHNOLOGY AND ASSOCIATED DEVICES IN THE FRAMEWORK OF SIMPLE THEORETICAL MODELING**" submitted by **Smt. Subhalaxmi Chakraborty**, who got her name registered on 18.11.2015 for the award of Ph. D. (Engg.) degree of Jadavpur University is absolutely based upon her own work under the supervision of **Dr. Chintan Kumar Mandal** and **Dr. Sankar Gangopadhyay** and that neither her thesis nor any part of the thesis has been submitted for any degree/diploma or any other academic award anywhere before.

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Abstract

Single-mode optical fiber, through which information is communicated via single mode, has emerged as the most effective medium of optical communication systems (OCS) in the context of broadband transmission. The operating wavelength of OCS is selected in the wavelength band ranging from 1.3 μm to 1.6 μm since the optical fiber made of silica is characterised by minimum attenuation loss to the extent of 0.15 dB/Km at the wavelength 1.55 μm and almost zero material dispersion at the wavelength 1.3 μm. Again, by shifting the zero dispersion wavelength to 1.55 μm region, one can obtain minimum attenuation loss as well as low dispersion simultaneously and this will lead to long repeater less transmission together with large band width. Fibers of this kind are known as dispersion-shifted fibers. Further, the operating wavelength of Erbium-doped fiber amplifier being 1.55 μm, the dispersion-shifted fiber happens to be an important device in the field of all optical technology. Moreover, in another type of fiber, known as dispersion-flattened fiber, almost zero dispersion is obtained over a wide range of wavelengths. Fiber of this type is used for increasing the information carrying capacity by the technique of wavelength division multiplexing. Therefore, the dispersion-shifted and dispersion-flattened fibers are of enormous importance in OCS as well as in various optical devices.

Thus, investigations of splice losses associated with single-mode dispersion managed fibers in the context of OCS have emerged as a potential problem. In addition, the performance of dispersion managed fibers as directional coupler, switches etc. can be investigated. Further, the presence of nonlinearity results in compression of pulse while dispersion causes broadening of pulse and thus interplay between dispersion and nonlinearity leads to propagation of optical beam as such. This is known as optical soliton, the study of which is extremely important in present OCS. Side by side, the study of dual-mode optical fiber in the context of optical fiber communication has also emerged as an interesting area of research. The large negative waveguide dispersion of dual mode optical fiber can be utilized to neutralise the positive dispersion and thus one can have broad band transmission of information through such fiber. Again, the study of the influence of nonlinearity on the fundamental modal field, the first higher order modal field and the associated characteristics will prove beneficial to the communication technologists in terms dispersion management and minimisation of modal noise as well.

Further, the study of the launch optics involving laser diode to the optical fiber coupling via microlenses on the fiber tip is also a potential problem in the area of OCS as well as associated devices. Accordingly, microlenses of various shapes as well as fibers of different refractive index profiles are being reported continuously in literature in order to maximise the source to fiber coupling efficiency. The objective is to identify the suitable refractive index profile of the fiber as well as the suitable microlens for maximum launch optics.

In chapter 1, the thesis presents the basics of fiber optics together with the relevant mathematical formulation by using electromagnetic theory. This chapter also describes briefly the technology of launch optics and other associated topics of interest. This chapter also contains extensive literature survey concerned with the present investigation. The objective and importance of the present work in the context of contemporary interest has also been presented and also the scope of the work in terms of its application in future research work has also been suggested.

In chapter 2, the series expression of fundamental modal field based on Chebyshev technique is employed to prescribe analytical expressions for splice losses in single-mode dispersion-shifted trapezoidal as well as dispersion-flattened graded and step W fibers. The formulated expressions are used to estimate the concerned losses in case of some typical fibers of the said kinds. It has been also shown that our results match excellently well with the available exact results, which are obtained by extensive computations. But, the execution of our formalism is simple and thus it is expected to benefit the system engineers.

In chapter 3, using the coupled-mode theory and the said Chebyshev power series expression for fundamental mode of dispersion managed fiber, we prescribe analytical formulation of normalised coupling length in terms of fiber to fiber separation for a directional coupler containing two identical single-mode dispersion managed fibers. The said estimations have been made for directional couplers corresponding to some typical dispersion-shifted trapezoidal as well as dispersion-flattened graded and step W fibers and the results obtained are found to be agreeing excellently with the available numerical results in case of directional coupler formed of two identical single-mode graded index fibers. The analysis is simple and as such it will prove used friendly to technologists who are working in the field of optical technology.

In chapter 4, we develop a simple iterative method involving Chebyshev formalism to predict the modal field of single mode graded index fiber both in presence and in absence of

Kerr-type nonlinearity. In chapter 5, we also report evaluation of first higher order modal field for dual- mode graded index optical fiber. Here also, the study is carried out both in absence as well as in presence of Kerr nonlinearity. The analyses in both chapters 4 and 5 are based on simple iterative method involving Chebyshev formalism. Taking some typical step and parabolic index fibers as examples in both cases, we show that our results match excellently with the available exact results obtained rigorously by applying finite element method. Thus the reported technique can be considered as an accurate alternative to the existing cumbersome techniques.

Chapter 6 deals with the conclusions derived from the investigations made. This chapter surfaces the merit of the investigation from the stand point of current research work.

The chapter 6 is followed by “Appendices” and “References”.

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CHAPTER-1

INTRODUCTION

1.1 BASICS OF FIBER OPTICS

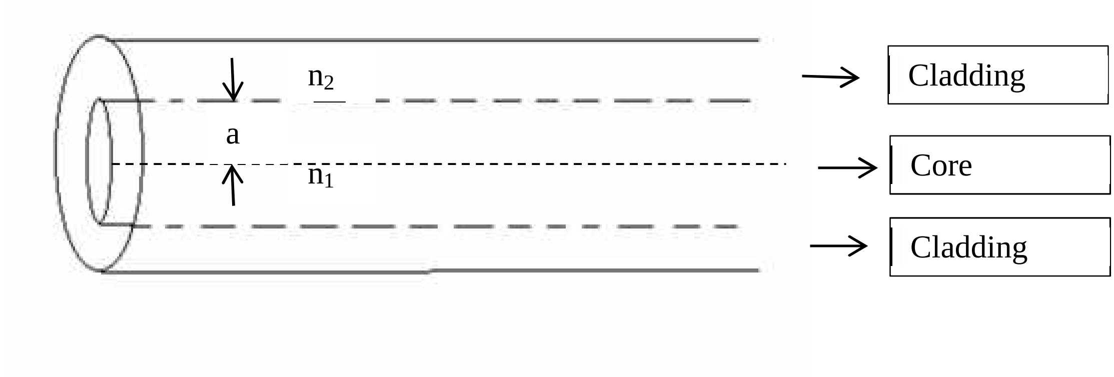

Fig.1.1 Step index fiber with core refractive index n_1, cladding refractive index n_2 and core radius a.

In the present age of information technology, optical fibers have emerged as the most important subject of research and development. Optical fibers are being extensively used in communication, signal processing, sensors etc. Optical fiber communication system is characterised by low loss and high bandwidth and this is why it has the ability to ensure efficient transmission of signal containing enormously large information traffic. An optical fiber basically contains a thin cylindrical dielectric core and the core is surrounded by a coaxial cylindrical dielectric shell which is known as cladding. The refractive index of cladding is kept slightly less than that of the core and the refractive index profile of the core is tailored suitably depending on its application. The fibers are named according to the nature of refractive index distribution inside the core. The fiber core having uniform refractive index inside the core is called step index fiber. There are other kinds of fibers like graded index fiber, trapezoidal fiber, graded W fiber, step W fiber etc., which possess nonhomogeneous refractive index distribution inside the core. The refractive index distribution inside core (n) versus normalised radial distance (R) of some typical fibers have been presented in the following figures. Here, R is equal to r/a, with r being the radial distance from the axis of the core and a being the core radius.

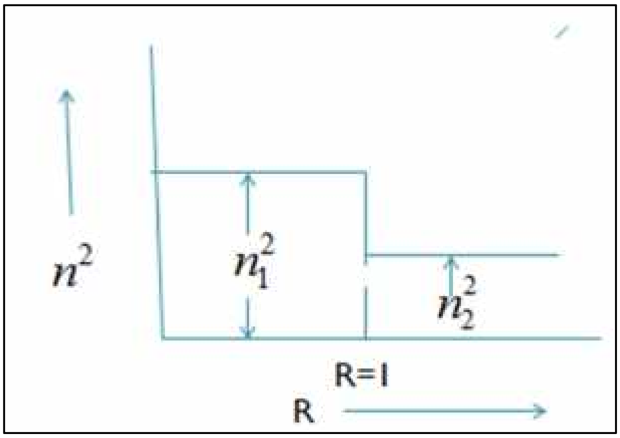

Fig. 1.2: Step index fiber

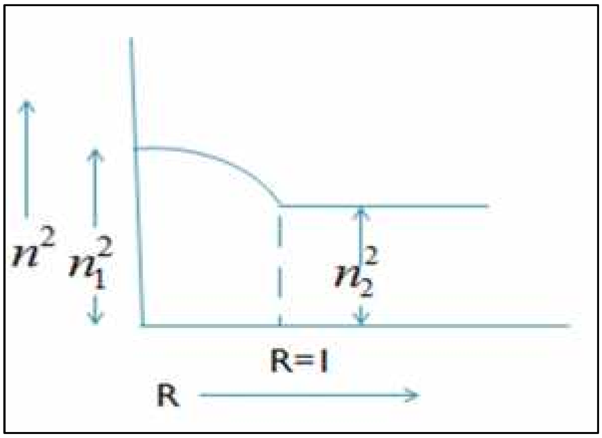

Fig. 1.3: Graded index fiber

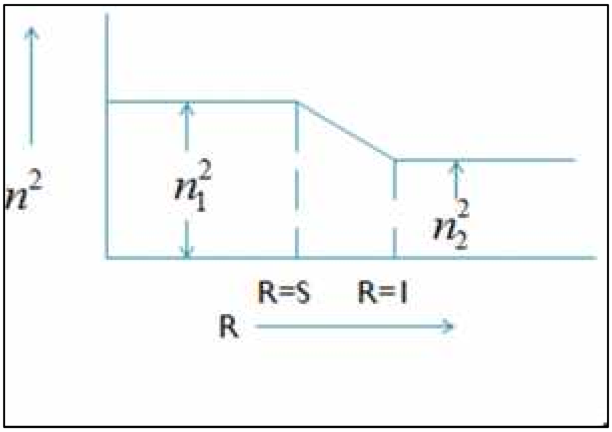

Fig. 1.4: Dispersion-shifted trapezoidal fiber

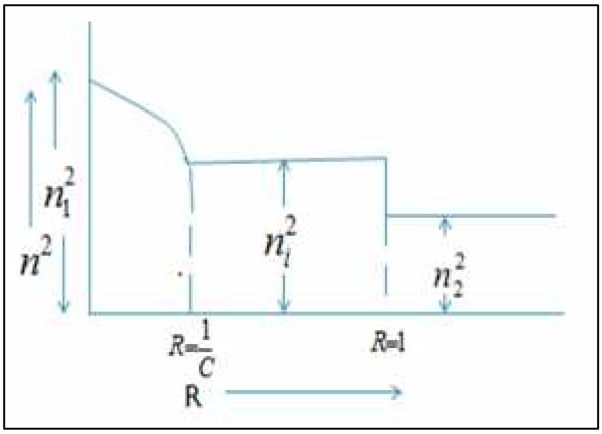

Fig. 1.5: Dispersion-flattened graded W fiber

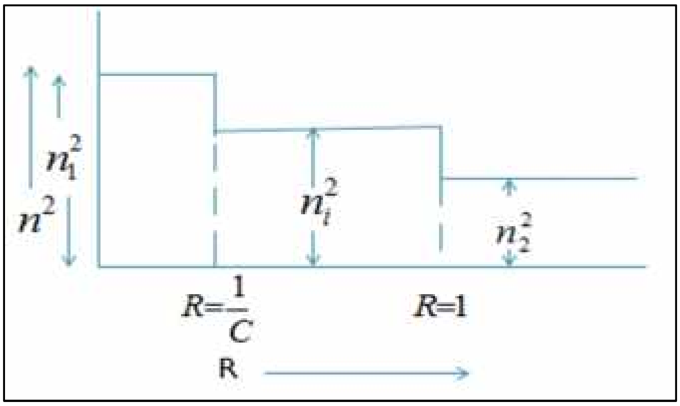

Fig. 1.6: Dispersion-flattened step W fiber

The fiber material is abundantly available silica which is doped with suitable elements like germanium oxide for creation of slight refractive index difference between core and cladding in order to minimise pulse dispersion and scattering loss as well (Neumann, 1988). The optical fiber is kept jacketed to protect it from damage due to external causes. The attenuation loss in optical fiber was due to impurity and its value was about 20dB/Km (Kapron, Keck and Maurer, 1970). Later purification technology advanced significantly and thus it became possible to reduce attenuation loss to nearly 0.15 dB/Km at the wavelength 1.55 µm (Kanamori, Yokota, Tanaka, Watanabe, Ishiguro, Yoshida, Kakii, Itoh, Asano, and Tanaka, 1986) and nearly 0.35dB/Km at the wavelength 1.3 µm (Jablonowski, 1986).

Optical pulses carrying information through optical fiber generate various modes inside the fiber. The physics of various modes can be understood by making electromagnetic analysis with the help of Maxwell’s equations. A mode can be defined as a specific transverse distribution of electromagnetic field and it is characterised by specific group velocity, state of polarisation and propagation constant as well. Taking into consideration the number modes supported by the fiber, the fiber is called single-mode or dual-mode or multimode. The fiber carrying the fundamental mode only is called single-mode fiber while the fiber carrying the fundamental as well as first higher order mode is called the dual-mode fiber. Besides these, there are multimode fibers supporting many modes inside them. Normally core diameter of single-mode fiber ranges between 5 and 10 µm while that of multimode fiber is around 50 µm. The cladding diameter is, however, around 125 µm in both kinds of fibers.

An important propagation parameter known as normalised frequency or V number characterises a fiber in terms of its mode support. Each mode is characterised by a cut-off V number below which it cannot propagate. Accordingly, V number of single-mode fiber is kept below the cut-off V number of first higher order mode so that propagation of only the fundamental mode is allowed inside the fiber. Optical communication systems are usually operated in the wavelength range from 1.3 to 1.6 µm since it has low attenuation loss (~ 0.15 dB/km) at the wavelength 1.55 µm and the material dispersion vanishes around the wavelength 1.3 µm. For long distance communication of large information with minimum number of repeaters, attenuation and dispersion are two important factors relating to the guidance of information through the fiber.

The dispersion causes broadening of a particular mode. The dispersion consists of three components namely waveguide dispersion, material dispersion and composite profile dispersion.

The waveguide dispersion is due to the dependence of propagation constant on the wavelength while material dispersion arises due to the dependence of the refractive indices of the core and cladding on the wavelength. The composite profile dispersion is proportional to the derivative of relative core-cladding refractive index difference with respect to wavelength. The value of the composite profile dispersion is less than 0.5ps/ (km nm) and thus it can be neglected from practical point of view. In case of multimode fiber, intermodal as well as intramodal dispersions are both present resulting in large broadening of optical pulse as it propagates through the fiber. Thus, multimode mode fiber is not suitable in the context of long distance communication of information. On the other hand, single-mode fiber, which permits guidance of only the fundamental mode, is free of intermodal dispersion. Thus, optical communication using single-mode fiber is more suitable in comparison to communication system using multimode fibers.

It can be mentioned in this connection that narrow spectral width of the light source used in communication system causes both material and waveguide dispersion leading to broadening of the pulse. In fibers made of silica, waveguide dispersion and material dispersion are of opposite signs in the wavelength range 1.3 to 1.8 μm. The fiber parameters can be judiciously selected so that the material dispersion nuetralises the waveguide dispersion at any wavelength of choice in the said wavelength band. In case of silica, the zero material dispersion occurs around 1.274 μm (Cohen, Lin and French, 1979; Neumann, 1988; Ghatak and Thyagarjan, 1998; Tian, Markov, Wang and Skorobogatiy, 2015) while the waveguide dispersion shifts the zero material dispersion wavelength towards longer wavelength. Usually, core radius of the fiber and the refractive index profile inside the core are suitably adjusted so that zero material dispersion wavelength is obtained at 1.55 μm (Cohen, Lin and French, 1979; Neumann, 1988; Ghatak and Thyagarjan, 1998). Fiber of this kind is called dispersion shifted fiber (Paek, 1983). This kind of fiber favours long repeater less communication having large bandwidth. Moreover, there is another type of fiber where almost nil dispersion is achieved over a range of wavelengths. Fiber of this kind is called dispersion flattened fiber (Mishra, Hosain, Goyal and Sharma, 1984) which enhances the information carrying capacity by wavelength division multiplexing (Olsson, Hegartz, Logen, Johnson, Walker, Cohen, Kasper and Campbell, 1985). Side by side, efforts are being made continuously in order to shape the refractive index profile in such a way that the first higher order cut-off V number is increased. This leads to large core radius and smaller splice loss.

Moreover, the study of dual-mode optical fiber has also generated tremendous interests (Spajer and Charquille, 1986; Eguchi, 2001; Eguchi, Koshiba and Tsuji, 2002; Amin, Ali, Chen and Shieh, 2011). In case of dual-mode optical fiber, the large negative waveguide dispersion associated with first higher mode can be used to nullify the positive dispersion. Thus, a dispersion compensated dual mode fiber can be designed so that it operates around the wavelength 1.55 μm which also happens to be the wavelength at which erbium-doped fiber amplifier usually works (Pedersen, 1994). It has been also found that double-layer profile core dispersion shifted fibers possess less bending and transmission losses in comparison to dispersion shifted fibers having simple core-cladding design (Monerie, 1982).

Therefore, a dual mode fiber with a double layer profile emerges as an important optical communication medium in terms of its low bending loss, low transmission loss and dispersion compensation around the wavelength 1.55 μm. It deserves mentioning in this connection that group delay between the first higher order mode and fundamental mode is utilised successfully in the technology of sensors. Moreover, method has been developed for separate evaluation of losses associated with fundamental and higher order mode (Ohashi, Kitayama, Kobayashi and Ishida, 1984). Again, excitation of dual-mode fiber by ultra-short laser pulse leads to generation of both the fundamental and the first higher order mode and this method has been utilised for study of various propagation characteristics associated with first higher order mode (Ohashi, Kitayama, Kobayashi and Ishida, 1984). Thus, study of dual mode fiber in presence as well as in absence of various kinds of nonlinear and doping elements, has generated tremendous interests.

The dispersion for graded index fiber is less compared to that for step index fiber. Thus, use of graded index fiber in optical communication enhances the bandwidth of transmission. Accordingly, prescription of simple but accurate formalism for prediction of propagation characteristics related to fundamental as well as first higher mode in different fibers is proliferating in literature. Again, development of general framework for study of light wave propagation through the optical fiber requires electromagnetic analysis. Thus, one needs to solve vector wave equations in order to obtain electric and magnetic field vectors. However, graded index fibers used for communication purpose are characterised by very small relative core cladding refractive index difference (less than 0.5%). Such fibers are called weakly guiding fibers (Neumann, 1988; Ghatak and Thyagarjan, 1998) and in case of such fibers, the vector wave equations can be approximated as scalar wave equations without sacrifice of accuracy. The said approximation is called weakly guiding approximation. In fact, it can be shown that for the said kinds of fibers, the propagation parameters found by using complicated vector wave equations differ insignificantly from those found from scalar wave equations (Gloge, 1971).

The analysis on the basis of scalar wave equation shows that modes in the fibers are almost linearly polarized with very small longitudinal components. It is relevant to mention in this context that analytical solution for scalar wave equation is available for step index fiber only. Besides step index fiber, one needs to apply either numerical or variational (Pattojoshi and Hosain, 1998; Chaudhuri and Roy, 2007; Khijwania, Nair and Sarkar, 2009; Ghosh, Roy and Bhadra, 2010; Behera, Hosain and Pattojoshi, 2011; Mallick and Sarkar, 2014) technique in order to obtain the solution of scalar wave equation.

The single parameter variational technique (Marcuse, 1978) provides analysis in a simple fashion but the results found lack accuracy. The double parameter variational technique permits the analysis in an accurate fashion but the concerned execution involves complicated computations (Hossain, Sharma and Ghatak 1982; Mishra, Hosain, Goyal and Sharma, 1984; Ankiewicz and Peng, 1992).

Thus, the literature of fiber optics requires expressions for fundamental as well as higher order modal fields for various kinds of fibers. The concerned prescription will allow one to predict the propagation parameters associated with each mode in case of each kind of fiber. Further, it can be mentioned in this connection, a simple power but accurate power series form of fundamental modal field of graded index fiber based on Chebyshev formalism is available in literature (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997). This method involves use of a linear formulation of $\frac{K_1(W)}{K_0(W)}$ with $\frac{1}{W}$ over a long and practical range of W values ($0.6 \le W \le 2.5$).

For the sake of simplicity, only the first four terms in the series expression for the fundamental mode of graded index fiber, is retained. Still, it has been shown that the modal fields evaluated by this simple formalism match excellently with the exact results found by cumbersome computations. Moreover, the application of this simple formulation for the purpose of estimation of different propagation parameters of single-mode graded index fibers has produced excellently accurate results (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997; Gangopadhyay and Sarkar, 1997a, 1998a, 1998b; Gangopadhyay, Choudhury and Sarkar, 1999).

The said formalism for prediction of fundamental mode of graded index fiber has also been extended to the concerned study in the low V region with some minor modification. In the low V region, linear least square fitting technique is applied in order to formulate linearly $\frac{K_1(W)}{K_0(W)}$ as a function of $\frac{1}{W}$ for different short intervals of W (Patra, Gangopadhyay and Sarkar, 2000).

It has also been shown that application of this simple series formulation appropriate for low V number has resulted in accurate prediction of different propagation parameters associated with graded index fiber of low V number like V<1.9 and V<1.4 for parabolic and step index fibers respectively (Patra, Gangopadhyay and Sarkar, 2001a; Patra, Gangopadhyay and Sarkar, 2001b).

The extension of this simple series expression for fundamental mode has also been made for study of dispersion shifted as well dispersion flattened fibers (Bose, Gangopadhyay and Saha, 2012c). The execution of this formalism is simple and the results found have been shown to be excellent. As mentioned earlier, the dual-mode fiber has also emerged as a potential medium of transmission of information and thus study of first higher order modal field is also an important matter. In this context, it can be mentioned that analytical expression for first higher order modal field is available only in case of step index fiber. Still, its execution for prediction of different propagation parameters related to first higher order mode involves lengthy computation owing to the presence of modified Bessel functions.

Existing numerical methods (Sharma, Goyal and Ghatak, 1981) for evaluation of first higher order modal field in case of graded index fiber, however, require rigorous computation involving a lot of time and as such these methods are not suitable from practical point of view.

The Chebyshev formalism for prediction of first higher order mode cut-off V number in case of graded index fiber is available in literature (Chen, 1982; Shijun, 1987). The concerned calculations require solution of an equation involving a third order determinant and the execution can be even made with the help a pocket calculator. Thus, this formulation will prove extremely advantageous to the system engineers. In this context, literature has been further enriched by formulation of power series expression for first higher order modal field of graded index fiber (Patra, Gangopadhyay and Goswami, 2008). This simple power series formulation has been found to be excellent in estimating different propagation parameters related to first higher order mode of graded index fiber (Bose, Gangopadhyay and Saha, 2011a; Bose, Gangopadhyay and Saha, 2011b; Bose, Gangopadhyay and Saha, 2012a).

Any optical communication system comprising single-mode optical fibers involves splice losses and thus literature requires prescription of simple but accurate formalism for prediction of transmission coefficients at the splice (Hosain, Sharma and Ghatak 1982; Gangopadhyay, Choudhury and Sarkar, 1999; Debnath and Gangopadhyay, 2016) in presence of practical transverse and angular mismatches. The necessary prediction, however, requires accurate knowledge of fundamental mode of concerned kind of fiber. Accordingly, one may be motivated to apply the simple series expression for fundamental mode for the needful prediction. Further, as regards study of optical devices like filters, fiber Bragg grating, directional couplers etc. (Lam and Garside, 1981; Gaylord and Moharam, 1985; Kersey, Berkoff and Morey, 1993; Jung, Nam, Lee, Byun and Kim, 1999; Sanyal, Gangopadhyay and Sarkar, 2000; Fang, Liao and Wang, 2010; Dai, Wang and Bowers, 2011), one may also be motivated to apply the said simple series formulation of the modal field.

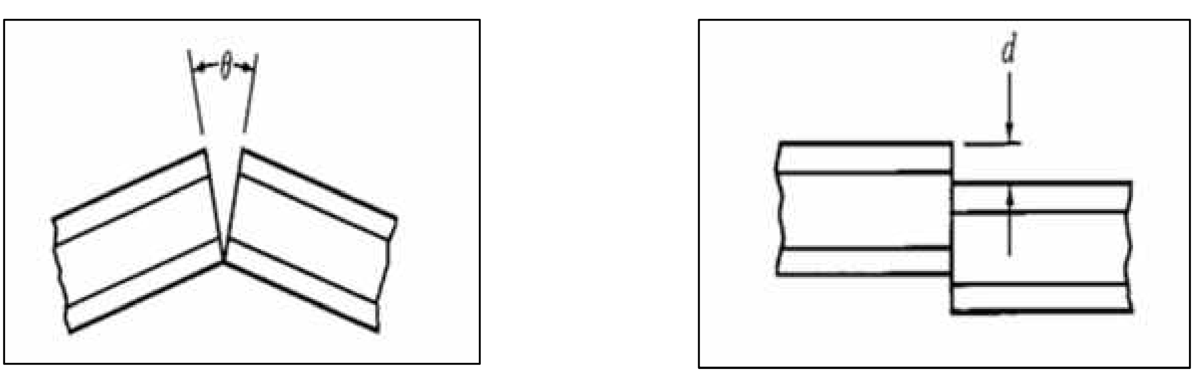

Fig. 1.7 (a) Angular mismatch ¯ at the splice; (b) Transverse mismatch d at the splice

Thus, the said Chebyshev based formalism for study of propagation parameters related to fundamental as well as first higher order mode in case of graded index dispersion managed fibers, requires more attention for study of related fiber optic components. Side by side, attempts are being constantly made in order to maximise laser source to fiber excitation efficiency.

Further, in the area of coupling optics, different kinds of microlenses on tips of fibers are being fabricated for maximum coupling efficiencies (Saruwatari and Nawata, 1979; Ghafouri-Shiraj and Asano, 1986; Sarkar, Pal and Thyagrajan, 1986; Ghafouri-Shiraj, 1988; Hillerich and Guttmann, 1989; Modavis and Webb, 1995; An, 2000; Rahman, Takahashi and Teik, 2002, 2003; Thual, Chanclou, Gautreau, Caledec, Guignard and Besnard, 2003; Liu, Liren, Rongwei and Zhu, 2005; Sambanthan and Rahman, 2005; Hu, Lin, Hung, Yang and chao, 2008; Liu, 2008; Chao, Hu, Hung and Yang, 2010; Huang and Yang, 2010; Yang, Chen, Ro and Liang, 2010; Mukhopadhyay, 2016). In this context, a microlens on the fiber tip will be most efficient coupler provided it has focal length which leads to matching of laser and fiber modes, it has aperture large enough to collect the entire input radiation and it is free of spherical aberration (Presby and Edwards, 1992; Edwards, Presby and Dragone, 1993). Incidentally, a hyperbolic microlens on the fiber tip satisfies all the said criteria and thus it can produce coupling efficiency around 100% at a particular focal length (Presby and Edwards, 1992). Moreover, the fabrication of hyperbolic microlens on the fiber tip requires sophisticated laser micromachining technique. Hemispherical microlens on account of mode mismatch, spherical aberration and limited aperture is not efficient in respect of coupling. But its fabrication on the fiber tip can be done by simple photolithographic technique and this is why it is being widely used as a coupler in spite of its slightly less coupling efficiency (Edwards, Presby and Dragone, 1993).

The evaluation of excitation efficiency on the basis of phase matching technique requires complicated numerical integrations. However, ABCD matrix (Massey and Siegman, 1969; McMullin, 1986; Yariv, 1991) technique has made the prediction of coupling optics simple but accurate (Gangopadhyay and Sarkar, 1996, 1997b, 1998c, 1998d, 1998e; Mukhopadhyay, Gangopadhyay and Sarkar, 2007, 2010; Huang and Yang, 2010; Mukhopadhyay and Sarkar, 2011). Side by side, tapered lenses of different profiles (Mondal, Gangopadhyay and Sarkar, 1998; Mukhopadhyay, Gangopadhyay and Sarkar, 2010) as well as suitable refractive index profiles inside fibers (Mukhopadhyay and Sarkar, 2011; Bose, Gangopadhyay and Saha, 2012b) are being designed in order to achieve maximum coupling efficiency. In this context, relevant ABCD matrix for each kind of device is formulated for the prediction of coupling optics in simple but accurate fashion. It is relevant to mention in this connection that modal spot size (around 5 μm) has to match with the small spot size (around 1 μm) of incident laser beam for maximum coupling efficiency. Suitable lensing system increases the size of laser beam spot size so that it becomes comparable to the fiber spot size. Accordingly, one can be motivated to carry on extensive investigations for prediction of cost effective coupler clubbed with prescription of user friendly formalism.

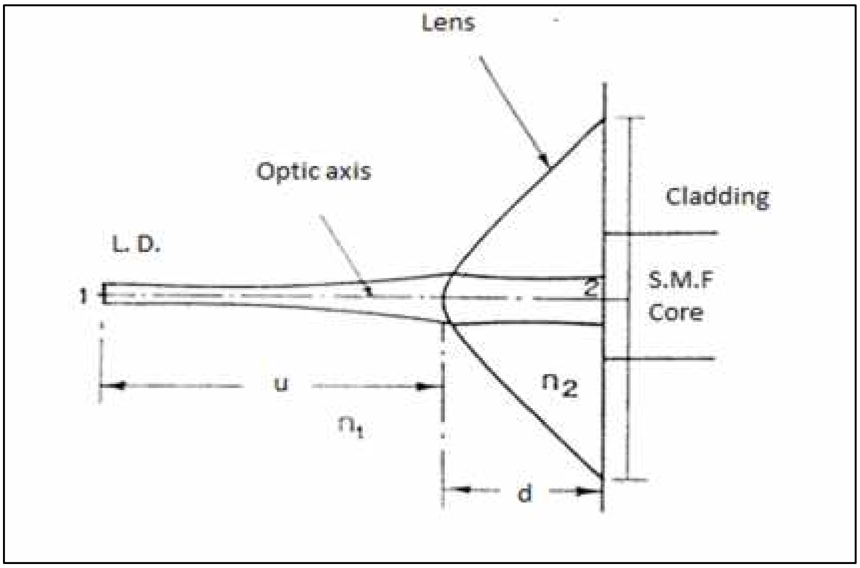

Fig 1.8: Schematic diagram of microlens on the tip of the fiber

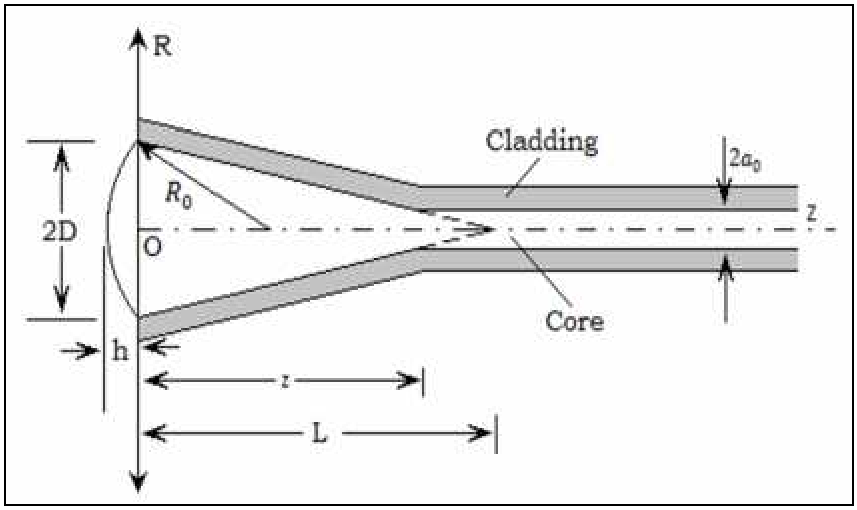

Fig. 1.9: Schematic diagram of tapered microlens drawn on optical fiber

In long haul communication system involving optical fibers, it is necessary to install optical amplifier between two communicating fibers for the purpose of amplifying the weak signal (Pedersen, 1994; Ghatak and Thyagarajan, 1999; Ono, Yamada and Shimizu, 2003; Thyagarajan and Kakkar, 2004a, 2004b). These optical amplifiers placed periodically, amplify the attenuated signals directly. Thus, the complicated process of conversion of optical signal to electrical signal, then amplification by electronic amplifier and finally restoring it to optical signal is avoided. Thus, the rare earth doped fiber amplifier emerges as a potential device in present optical communication system.

In this context, the efficient performance of such amplifier demands extensive study in terms of signal wavelength as well as doping material. As an optical amplifier, Raman gain amplifier has also shown its merit (Felinskyi and Dyriv, 2016; Tan, Rosa, Le, Iqbal, Phillips and Harper, 2016). Thus, there is enough scope for prescription of new design as well as concerned simple but accurate formalism for study of fiber amplifier.

Another emerging area is the study of propagation characteristics of optical fiber in presence of nonlinearity (Tomlinson, Stolen and Chank, 1984; Tai, Tomita, Jewell and Hasegawa, 1986; Snyder, Chen, Poladian and Mitchel, 1990; Goncharenko, 1990; Agrawal and Boyd, 1992). Nonlinearity can be catagorised as third order, fifth order etc. including saturable nonlinearity (Saitoh, Fujisawa, Kirihara and Koshiba, 2006). Intensity of optical beam as well as the nature of doping material, decide which kind of nonlinearity will be generated (Agrawal, 2013). The presence of nonlinearity causes pulse compression while dispersion broadens the pulse. Thus, suitable interplay between dispersion and nonlinearity leads to transmission of optical pulse as such (Agrawal and Boyd, 1992). This is known as propagation of temporal soliton (Herr, Brasch, Jost, Wang, Kondratiev, Gororodetsky and Kippenberg, 2014), the study of which has generated huge interest in the field of fiber optic communication as well as integrated photonics (Lu, Lee, Rogers and Lin, 2014). Another interesting area developed by nonlinearity is the study of spatial soliton (Sammut, Li and Pask, 1992; Gangopadhyay and Sarkar, 2001; Zhang, Huo and Duan, 2016) where diffraction effect is neutralized by change of refractive index profile due to nonlinearity. Thus, propagation of spatial soliton takes place spatially in undistorted form. Moreover, spatial and temporal solitons combined together generate light bullets which travel without distortion temporally as well as spatially (Yeh, 1994; Synder and Mitchell, 1997; Mihalache, 2012; Biswas, Mirzazadeh, Eslami, Zhou, Bhrawy and Belic, 2016). Thus, investigations relating to different propagation parameters of spatiotemporal solitons (Sharka, Berezhiani and Miklaszewski, 1997; Xu, Jovanoski, Bouasla, Triki, Moraru and Biswas, 2013; Zhou, Mirzazadeh, Zerrad, Biswas and Belic, 2016) have also emerged as potential area of research.

1.2 ELECTROMAGNETIC ANALYSIS OF OPTICAL FIBER

In case of a linear, isotropic, non-conducting, non-magnetic fiber material (Ghatak and Thyagarjan, 1999), Maxwell's equations for the fiber material can be given by

$$\vec{\nabla} \times \vec{E} + \mu_0 \frac{\partial \vec{H}}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{1.1}$$

$$\vec{\nabla} . \vec{B} = 0$$

$$\vec{\nabla} . \vec{D} = 0$$

where, $\vec{E}$, $\vec{H}$ and $\vec{B}$ represent the electric intensity, magnetic intensity and magnetic induction vectors respectively. Here, $\vec{D}$ represents the electric displacement vector which is given as

$$\vec{D} = \varepsilon_0 n^2 \vec{E} \tag{1.2}$$

where μ_0, ε_0 and n stand for the magnetic permeability of free space, the permittivity of free space and the refractive index of the corresponding medium respectively. Use of Eqs. (1.1) and (1.2) lead to the following vector wave equations

$$\nabla^2 \vec{E} + \vec{\nabla}\left(\vec{E} . \frac{\vec{\nabla} n^2}{n^2} \right) - \mu_0 \varepsilon_0 n^2 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{1.3}$$

$$\nabla^2 \vec{H} + \frac{1}{n^2}\left(\vec{\nabla} n^2\right) \times \left(\vec{\nabla} \times \vec{H}\right) - \mu_0 \varepsilon_0 n^2 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \tag{1.4}$$

In case of inhomogeneous medium, Eq. (1.3) shows that equations for E_x, E_y and E_z are coupled while Eq. (1.4) shows that equations for H_x, H_y and H_z are also coupled.

Further, we consider a refractive index profile which possesses translational invariance along the direction of propagation (Z axis) of a particular optical waveguide. Thus, we can write

$$n^2 = n^2(x, y) \qquad (1.5)$$

Accordingly, the time and Z dependent parts of electric (or magnetic) field should be of the form exp (±iωt) and exp (±iβz) respectively.

Thus, in case of such refractive index profile, the solutions of Eqs. (1.3) and (1.4) can be given as

$$\vec{E}\left(x, y, z, t\right) = \vec{E}\left(x, y\right) e^{j(\quad z-\mu\ t)}$$

$$= \left[\vec{E}_t\left(x, y\right) + \vec{E}_z\left(x, y\right)\right] e^{j(\quad z-\mu\ t)} \qquad (1.6)$$

$$\vec{H}\left(x, y, z, t\right) = \vec{H}\left(x, y\right) e^{j(\quad z-\mu\ t)}$$

$$= \left[\vec{H}_t\left(x, y\right) + \vec{H}_z\left(x, y\right)\right] e^{j(\quad z-\mu\ t)} \qquad (1.7)$$

Here, $\vec{E}_t$ (or $\vec{H}_t$) and $\vec{E}_z$ (or $\vec{H}_z$) are the transverse and longitudinal components of electric (or magnetic) field vectors respectively in the Cartesian coordinates system while represents the propagation constant.

Employing Eq. (1.6) in Eq. (1.3), one can obtain the vector wave equation for $\vec{E}_t$ as follows

$$\nabla_t^2 \vec{E}_t + \left[k_0^2 n^2(x, y) - \quad ^2\right]\vec{E}_t = -\vec{\nabla}_t.\left[\vec{E}_t.\vec{\nabla}_t\left(\ln n^2\right)\right] \qquad (1.8)$$

where,

$$\vec{\nabla}_t = \vec{\nabla} - \hat{z}\frac{\partial}{\partial z}; \qquad k_0 = \mu / c = 2® / ª_0 \tag{1.9}$$

Here, k_0 represents the free space wave number.

In order to get minimum pulse dispersion and scattering loss simultaneously, the manufacturers of optical fibers maintain small relative core cladding refractive index difference. This condition is termed as weakly guiding condition. This condition, which is also known as scalar wave approximation, allows one to neglect the term on the right hand side of Eq. (1.8). Accordingly, Eq. (1.8) reduces to

$$\nabla_t^2 f + \left[k_0^2 n^2(x, y) - \;^2\right] f = 0 \tag{1.10}$$

The x or y component of $\vec{E}(x, y)$ is denoted by f while ∇_t^2 represents the scalar Laplacian operator.

Further, since the optical fiber has cylindrical symmetry, the refractive index depends on the radial coordinate only and thus it is convenient to write $n^2(x, y) = n^2(r)$ and use cylindrical coordinates for the necessary electromagnetic analysis of the fiber.

Accordingly, Eq. (1.10) can be written as

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial ¤^2} + \left[k_0^2 n^2(r) - \;^2\right] f = 0 \tag{1.11}$$

We can solve Eq. (1.11) by using separation of variables method since the medium has cylindrical symmetry. Thus, we write

$$f(r, ¤) = \cdot\,(r) F(¤) \tag{1.12}$$

Employing Eq. (1.12) in Eq. (1.11), we have

$$\frac{r^2}{\psi}\left(\frac{d^2\psi}{dr^2}+\frac{1}{r}\frac{d\psi}{dr}\right)+r^2\left[k_0^2n^2(r)-\beta^2\right]=-\frac{1}{F}\frac{d^2F}{d\phi^2}=l^2 \tag{1.13}$$

Here, l is a constant.

Eq. (1.13) leads to the following relation

$$\frac{d^2F}{d\phi^2}+l^2F=0 \tag{1.14}$$

The solution of Eq. (1.14) can be given as

$$F(\phi) = \sin l\phi \;\; or \;\; \cos l\phi \tag{1.15}$$

The corresponding boundary condition $F(\phi+2\pi)=F(\phi)$ gives $l=$ 0, 1, 2…etc., where the constant l is called the azimuthal mode number. It deserves mentioning in this connection that the negative values of l correspond to the same field distribution.

Further, Eq. (1.13) also gives the following equation for $\psi(r)$

$$\frac{d^2\psi}{dr^2}+\frac{1}{r}\frac{d\psi}{dr}+\left[k_0^2n^2(r)-\beta^2-\frac{l^2}{r^2}\right]\psi=0 \tag{1.16}$$

For a given value of l and a prescribed set of values of the fiber parameters, Eq. (1.16) leads to a finite number of allowed solutions for β^2 which can be presented as β_{l1}^2, β_{l2}^2, β_{l3}^2,.........etc. Thus the general representation of those can be written in the form of β_{lm}^2 with m being 1, 2, 3,….etc. The corresponding modes are known as LP_{lm} (linearly polarized) modes while ‘m’ is termed as radial mode number. For example, β_{11} stands for the propagation constant corresponding to first solution of Eq. (1.16) for l =1. The corresponding mode is known as LP_{11} mode. Here, the term LP denotes linearly polarized mode.

In case of weakly guiding fiber, the refractive index profile is given by

$$n^2(R) = n_1^2(1 - 2\Delta\, f(R)), \qquad R \le 1$$

$$n^2(R) = n_2^2, \qquad R > 1 \tag{1.17}$$

where, $R = \dfrac{r}{a}$, a = core radius, $\Delta = (n_1^2 - n_2^2)/2n_1^2$ with n_1 and n_2 being the refractive indices of the core axis and the cladding respectively.

The shape of the refractive index profile is represented by $f(R)$.

Eqs. (1.16) and (1.17) lead to the following equation

$$\frac{d^2\psi}{dR^2} + \frac{1}{R}\frac{d\psi}{dR} + \left[V^2\left(1 - f(R)\right) - W^2\right]\psi - \frac{l^2}{R^2}\psi = 0, \qquad R \le 1 \tag{1.18}$$

$$\frac{d^2\psi}{dR^2} + \frac{1}{R}\frac{d\psi}{dR} - W^2\psi - \frac{l^2}{R^2}\psi = 0, \qquad R > 1 \tag{1.19}$$

Here, $V\left(= k_0 a\left(n_1^2 - n_2^2\right)^{1/2}\right)$ is known as the normalized frequency or the dimensionless frequency or simply V number of the fiber and $W\left(= a\left(\beta^2 - k_0^2 n_2^2\right)^{1/2}\right)$ is the cladding decay parameter.

In case of fundamental mode ($l = 0$), Eqs. (1.18) and (1.19) are respectively given by

$$\frac{d^2\psi}{dR^2} + \frac{1}{R}\frac{d\psi}{dR} + \left[V^2\left(1 - f(R)\right) - W^2\right]\psi = 0, \qquad R \le 1 \tag{1.20}$$

$$\frac{d^2\cdot}{dR^2}+\frac{1}{R}\frac{d\cdot}{dR}-W^2\cdot\ =0,\qquad R>1 \tag{1.21}$$

For large value of WR, the solution of Eq. (1.21) can be approximated as

$$\cdot\ (R)\sim\left(\frac{\text{®}}{2WR}\right)^{1/2}\exp(-WR) \tag{1.22}$$

Eq. (1.22) shows that the electric field inside the cladding decays almost exponentially as a function of WR. Accordingly, W is called the cladding decay parameter.

Moreover, for a particular refractive index profile, the number of allowed values of , which gives the number of LP modes propagating through the fiber, depends on ‘a’, ‘n_1’, ‘n_2’ and a and thus on the normalized frequency ‘V’.

The propagation constant for guided mode in optical fiber, which decays in an exponential manner inside the cladding, must satisfy the following condition

$$k_0 n_2 < \quad < k_0 n_1 \tag{1.23}$$

It is clearly seen that in case $\quad \le k_0 n_2$, the mode is cut off and this mode leaks into cladding instead of being guided through core. Such modes are known as leaky modes.

1.3 OBJECTIVE AND SCOPE OF THE THESIS

Dispersion-shifted as well as dispersion-flattened optical fibers have emerged as an important media in the field of optical technology. The design of optical communication system involving such dispersion managed fibers requires accurate knowledge of splice losses. In this context, the fundamental modal field associated with each kind of said fibers has to be known accurately. Moreover, it is desirable that the expression for the field should be simple so that concerned evaluations can be executed easily with sufficient accuracy.

Literature has been already enriched with the Chebyshev power series expressions for fundamental modes of dispersion-shifted as well as dispersion-flattened fibers. The simplicity and the accuracy of the power series expression in predicting different propagation parameters associated with such dispersion managed fibers have motivated us to employ the concerned power series expressions for evaluation of transmission coefficient at the splice in presence of both transverse and angular mismatches. It may be mentioned in this connection that splices being highly tolerant with respect to longitudinal separation, the investigations have been restricted to transverse and angular mismatches only.

Directional coupler, consisting of two adjacent, parallel single mode optical fibers, use evanescent field coupling. It has emerged as the most prospective device in the domain of sensors, switches, filters etc.. The design of directional coupler requires knowledge of coupling parameters relevant for particular field of use. Being motivated by the simplicity and accuracy of the series expression for fundamental mode of each kind of dispersion managed fiber, we use the coupled-mode theory together with series expression for fundamental mode in order to predict the coupling characteristics of directional coupler made of two identical types of dispersion managed fibers in a very simple but accurate manner.

Again, the power series formulation for the fundamental mode for graded index fiber by Chebyshev technique has been found to be excellent in predicting accurately different propagation parameters in simple fashion. This led us to develop a simple iterative method involving the said power series expression in order to predict the modal field of single-mode graded index fiber both in presence and absence of Kerr-type nonlinearity.

Further, dual-mode optical fiber has also emerged as a potential candidate in the field of optical fiber communication system. Moreover, Chebyshev power series expression for first higher order modal field in case of graded index fiber is available in literature. The said power series expression has produced excellent results in predicting different propagation parameters associated with first higher order mode in case of linear graded index fibers. Taking into consideration that evaluation of first higher order modal field associated with Kerr-type nonlinearity is also an important matter in the study of dual-mode fiber, we develop an iterative technique in the context, using the Chebyshev power series expression for first higher order modal field of graded index fiber.

The excellent prediction of splice losses as well as coupling characteristics of directional coupler by this simple formalism, generates ample scope for estimation of the said parameters involving fibers of different kinds.

Our results for both fundamental as well as first higher order modal fields for graded index fiber in presence of nonlinearity, match excellently well with the available exact results obtained by finite element method quite rigorously. Thus, the investigations involve formulation of some simple but accurate methods which will benefit the technologists. The execution of our formalism for evaluation of the concerned parameters in all the cases is very simple and as such it is user friendly. Further, the study made, leaves scope for extension of the formalism in devices involving photonic crystal fibers, holey fibers etc. Side by side there is scope for extension of our formalism in investigation involving different kinds of fibers having different types of nonlinearity.

1.4 STRUCTURE OF THE THESIS

The chapter of Introduction presents literature survey concerned with the review of the background and current scenario as well in the context of motivation of present work. The investigations made, have been described in the next four chapters. Finally, in chapter 6, the conclusion relating to the investigations associated with the research work has been presented. The thesis comprises three parts. The first part of the thesis has been presented in chapter 2, where the thesis involves prescription of a simple but accurate method for prediction of splice loss in single-mode dispersion- shifted trapezoidal as well as dispersion-flattened graded and step W fibers. Further, splice being highly tolerant for longitudinal separation, the analysis of splice loss relating to single-mode dispersion managed fiber in chapter 2 is judiciously restricted to the case of transverse and angular mismatches only. Chapter 3 contains the second part of the thesis where a novel, simple but accurate method has been developed for analysis of directional coupler made of dispersion managed fibers. The third part of the thesis is presented in chapters 4 and 5. Chapter 4 contains prediction of fundamental modal field for graded index fiber both in presence as well as in absence of Kerr-type nonlinearity while chapter 5 deals with prediction of first higher order modal field for graded index fiber both in presence and in absence of Kerr-type nonlinearity.

Regarding the estimation of splice loss given in first part of the thesis, the series expression for fundamental modal field appropriate for each kind of dispersion managed fiber is employed. It deserves mentioning in this connection that this method uses a linear relation of the ratio of first and zero-order modified Bessel function with reciprocal of cladding decay parameter. The cladding decay parameter is estimated from an equation consisting of a fourth order determinant (Bose, Gangopadhyay and Saha, 2012c). Simple but accurate analytical expressions for splice losses are formulated. Choosing some typical dispersion-shifted trapezoidal and dispersion-flattened graded and step W fibers, it has been shown that the concerned estimations match excellently with the available exact results. Further, the evaluations based on the prescribed formulations require little computations. Thus, this simple but accurate formalism is expected to be of immense importance in the field of optical technology.

As done in chapter 2, we, in chapter 3 also, applied the series expression for fundamental mode appropriate for each kind of dispersion-managed fiber. Analytical expressions for coupling characteristics of directional coupler involving two identical dispersion-managed fibers are prescribed. From the formulated expressions, one can easily select the normalised separation between the axes of the two fibers belonging to the directional coupler so that the coupling length remains less than 9.5 mm. The estimation is simple and as such it will prove user friendly to the technologists who are working with such devices.

In chapter 4, we have prescribed a simple but accurate method based on iteration in order to predict the modal field of single-mode graded index fiber in the presence as well as in absence of Kerr-type nonlinearity. Here, we have used the series expression for fundamental mode of graded index fiber developed by Chebyshev formalism and applied the iteration technique on it for obtaining the modal field of single-mode graded index fiber in presence as well as in absence of Kerr-type nonlinearity. The accuracy of this simple formalism may lead one to reconsider the prescribed method as an accurate alterative to the existing cumbersome finite element method. The execution of the formalism being simple, it will benefit the system users in the process of minimization of modal noise due to nonlinearity.

In chapter 5, we present evaluation of first higher order modal field for dual mode graded index fiber in the presence as well as in absence of Kerr-type nonlinearity. The analysis uses the simple series expression for first higher order modal field in case of graded index fiber. The said series expression has been formulated by Chebyshev method and the analysis both in presence and in absence of Kerr-type nonlinearity employs iterative technique involving the said series expression. The execution of the formalism is very simple but the results found match excellently well with the available exact results found by cumbersome finite element method. Thus, the formalism developed may prove immensely important in the field of communication, sensor etc. concerned with dual mode fiber.

In chapter 6, the conclusion relating to the research work has been presented in nutshell. There is also “Appendices” section following chapter 6. Mathematical formulae used in the research work have been presented in the section, ‘Appendices’. Finally, there is “References” section containing the published papers and books wherefrom citations have been made in our investigations. Further, the list of publications in the context of present work have also have also been presented in the page just before the page containing the certificate from the supervisors.

CHAPTER – 2

A SIMPLE BUT ACCURATE METHOD FOR PREDICTION OF SPLICE LOSS IN SINGLE-MODE DISPERSION-SHIFTED TRAPEZOIDAL AS WELL AS DISPERSION-FLATTENED GRADED AND STEP W FIBERS

2.1 INTRODUCTION

Presently, single-mode optical fiber has emerged as an effective broad band medium of communication. The optical fiber made of silica possesses minimum attenuation loss to the extent of 0.15 dB/km at the wavelength 1.55 μm while its material dispersion becomes zero at the wavelength 1.3 μm. This is the reason why long haul optical communication system operates at wavelengths ranging from 1.3 to 1.6 μm. Further, if the wavelength for zero dispersion is shifted to 1.55 μm, one achieves minimum attenuation loss and low dispersion simultaneously. This provides large bandwidth system having appreciably long repeater less transmission. Such fibers, known as dispersion-shifted fibers, can be produced by suitable change of the parameters of the fiber (Paek, 1983; Ainslie and Day, 1986; Tewari, Pal and Das, 1992). In view of the fact that Erbium-doped fiber amplifier operates at wavelength 1.55 μm, the dispersion-shifted fiber emerges as a potential device (Pederson, 1994; Thyagarajan and Kakkar, 2004a). Moreover, there is another type of fiber, where waveguide dispersion and the material dispersion almost cancel out with each other over a range of wavelengths. Fibers of this kind are called dispersion-flattened fibers, which can be employed to enhance the information carrying capacity by the technique of wavelength division multiplexing (Olsson, Hegartz, Logen, Johnson, Walker, Cohen, Kasper and Campbell, 1985). In this context, the prescription of simple but accurate expressions for fundamental modes of dispersion-shifted trapezoidal (Paek, 1983) and dispersion -flattened graded W (Mishra, Hosain, Goyal and Sharma, 1984) and step W fibers (Monerie, 1982; Garth, 1989) is of tremendous importance for predicting the various propagation characteristics of such fibers and also the coupling optics concerned with different kinds of lens systems. In this connection, it deserves mentioning that the generalised study of transmission of light through an optical fiber requires an electromagnetic analysis which involves tedious solution of vector wave equations. Now fibers, used for communication purposes, are made of low relative core-cladding refractive index difference (less than 0.5%). For such kind of fiber, called weakly guiding fiber, the vector wave equation can be reduced to scalar wave equation by reasonable approximation. It has been also established that the relevant parameters estimated by scalar and vector wave equations are almost identical and hence for the sake of simplicity and accuracy as well (Hosain, Goyal and Ghatak, 1983), it is justified to make the analysis by scalar wave theory. Still, prediction of propagation parameters for mono-mode weakly guiding fiber remains difficult due to the involvement of Bessel functions. The application of two parameter variational technique (Mishra, Hosain, Goyal and Sharma, 1984; Ankiewicz and Peng, 1992) can predict the fundamental mode and the associated propagation parameters with sufficient accuracy. But the concerned evaluations by this formalism require complicated computations. Therefore, formulation of simple but accurate expression of fundamental mode in this context is still in demand in literature. A simple power series expression of fundamental mode of graded index fiber based on Chebyshev technique is available in literature (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997). This simple form of fundamental mode has been shown to have estimated the propagation parameters of single-mode graded index fibers excellently (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997; Gangopadhyay and Sarkar, 1997a, 1998a, 1998b). This method comprises formulation of linear relation of $\frac{K_1(W)}{K_0(W)}$ with 1/W in the range defined by $0.60 \le W \le 2.5$. Following this simple formalism, simple but accurate expressions for fundamental mode of dispersion-shifted trapezoidal as well as of dispersion-flattened graded and step W fibers have also been prescribed (Bose, Gangopadhyay and Saha, 2012c). It has been shown that application of this formalism has led to accurate estimations of the relevant parameters in a simple fashion (Bose, Gangopadhyay and Saha, 2012c). The design of optical communication system involving such dispersion-shifted and dispersion-flattened fibers requires accurate knowledge of splice losses. For evaluation of transmission coefficients at the splice for fundamental modal field, one must know accurately such field for the said kinds of fibers. The accurate predictions of fundamental modal fields and cladding decay parameters of the dispersion managed fibers by the simple but accurate formalism prescribed in (Bose, Gangopadhyay and Saha, 2012c) have motivated us to apply this methodology in predicting transmission coefficients at the splice with angular offset and transverse offset in case of such fibers.

In this chapter, we report the prescriptions of analytical expressions of the said splice losses by this formalism and estimations of those thereof. We also show that our estimations are virtually indistinguishable from the numerical exact ones. In appendix A, we present how the series expression of fundamental mode for both dispersion-shifted as well as dispersion-flattened fiber has been formulated. The evaluations by this simple method, however, comprise some differentiation and integration involving modified Bessel functions (Watson, 1944; Gradshteyn and Ryzhik, 1980; Abramowitz and Stegun, 1981). In the appendix B, we have presented some important formulae regarding differentiation and integration of Bessel functions which have been used by us in the analysis. The formulae given are expected to benefit the system engineers in many branches of technology.

2.2. THEORY

The refractive index profile of optical fiber can be given by

$$n^2(R) = \begin{cases} n_1^2 (1 - 2\Delta f(R)), & R \le 1 \\ n_2^2 \ , & R > 1 \end{cases} \tag{2.1}$$

where, R = r/a with a being the radius of the core and $\Delta = (n_1^2 - n_2^2)/2n_1^2$ with n_1 and n_2 being the refractive indices of the fiber-core and the fiber-cladding respectively. Here, *f(R)* defines the shape of refractive index profile.

The profile functions *f(R)* for the concerned fibers are given below,

(I) $f(R) = 0 \ , \quad 0 < R \le S$

$f(R) = \dfrac{R-S}{1-S} \ , \quad S < R \le 1$

► dispersion-shifted trapezoidal fiber

(II) $f(R) = {}^\circ R^q \ , \quad R \le \dfrac{1}{C}$

$f(R) = {}^\circ \ , \quad \dfrac{1}{C} < R \le 1$

► dispersion-flattened graded W fiber

(III) $f(R) = 0 \ , \quad R \le \dfrac{1}{C}$

$f(R) = {}^\circ \ , \quad \dfrac{1}{C} < R \le 1$

► dispersion-flattened step W fiber

Here, S represents the aspect ratio for trapezoidal fiber. Again, q stands for profile exponent in case of W fiber and its value is ∞ for step type profile while ρ denotes the relative index depth of inner cladding having refractive index n_i and it is defined as, $° = \frac{(n_1^2 - n_i^2)}{(n_1^2 - n_2^2)}$.

The refractive index profiles of trapezoidal, graded W and step W fibers have been presented in Figs. 1.4, 1.5 and 1.6 respectively. Further, angular mismatch as well as transverse mismatch at the splice have been shown in Figs 1.7 (a) and 1.7(b) respectively.

In case of small angular mismatch θ, the concerned overlap integral is given as (Hosain, Sharma and Ghatak, 1982; Gangopadhyay, Choudhury and Sarkar, 1999)

$$C_a(p) = \int_0^{2®} d¤ \int_0^{\infty} R\,dR\ \left|\cdot\ (R)\right|^2 \exp(ipR\cos¤) \tag{2.2}$$

where, $p = ak_0 n_2 \overline{\ }$ with k_0 being the free-space wave number. It deserves mentioning in this connection that the fibers are assumed to be joined by an index matching fluid of refractive index n_2 . The transmission coefficient $T_a(p)$ at the splice with angular misalignment is defined as

$$T_a(p) = \left|\frac{C_a(p)}{C_a(0)}\right|^2 \tag{2.3}$$

Expanding the exponential term first, we obtain Eq. (2.2) as

$$C_a(p) = 2® \sum_{n=0}^{n=\infty} \frac{(-p^2/4)^n}{(n!)^2} \int_0^{\infty} \left|\cdot\ (R)\right|^2 R^{2n+1} dR \tag{2.4}$$

It has been shown in (Hosain, Sharma and Ghatak, 1982; Gangopadhyay, Choudhury and Sarkar, 1999) that for sufficient accuracy, one needs to retain only the first four terms in Eq. (2.4) for angular mismatch up to 1° which nearly corresponds to p= 0.8.

Using Eqs.(2.3) and (2. 4) and also, the expression for $\psi(R)$ given by Eq. (A13) in Appendix A together with the results (Watson, 1944; Gradshteyn and Ryzhik, 1980; Abramowitz and Stegun, 1981) given in Appendix B, we obtain (Gangopadhyay, Choudhury and Sarkar, 1999)

$$T_a(p) = (1 - S_1 p^2 / S_0 + S_2 p^4 / S_0 - S_3 p^6 / S_0) \tag{2.5}$$

where,

$$S_0 = [T_1 + T_2(T_3^2 - 1)]$$

$$S_1 = 0.5 p^2 [T_4 + (T_2/3)(T_3^2(0.5 + 1/W_c^2) + T_3/W_c - 0.5)]$$

$$\begin{aligned} S_2 &= (p^4/32)[0.5T_5 + T_2(T_3^2(4/15W_c^2 + (1/60)(1+8/W_c^2)^2 + 1/12) \\ &+ T_3(2/5W_c + 16/5W_c^3) + 4/15W_c^2 - 0.1)] \end{aligned} \tag{2.6}$$

$$\begin{aligned} S_3 &= (p^6/1152)[0.5T_6 + (T_2/7W_c^2)(T_3^2(72/5W_c^2 + 0.9(1+8/W_c^2)^2 + W_c^2/2 + 4.5) + \\ & T_3(288/5W_c^3 + 108/5W_c + 3W_c) + (14.4/W_c^2 - W_c^2/2 + 3.6))] \end{aligned}$$

with

$$T_1 = 1 + A_2^2/3 + A_4^2/5 + A_6^2/7 + 2(A_2/2 + A_4/3 + A_6/4 + A_2A_4/4 + A_2A_6/5 + A_4A_6/6)$$

$$T_2 = (1 + A_2 + A_4 + A_6)^2$$

$$T_3 = 1.034623 + 0.3890323 / W_c$$

$$T_4 = (0.25 + A_2^2/8 + A_4^2/12 + A_6^2/16 + A_2/3 + A_4/4 + A_6/5 + A_2A_4/5 + A_2A_6/6 + A_4A_6/7)$$

$$T_5 = 1/3 + A_2^2/5 + A_4^2/7 + A_6^2/9 + 2(A_2/4 + A_4/5 + A_6/6 + A_2A_4/6 + A_2A_6/7 + A_4A_6/8) \tag{2.7}$$

$$T_6 = 0.25 + A_2^2/6 + A_4^2/8 + A_6^2/10 + 2(A_2/5 + A_4/6 + A_6/7 + A_2A_4/7 + A_2A_6/8 + A_4A_6/9)$$

Here, W_c is the value of cladding decay parameter W (Nuemann, 1988) evaluated by the present method.

The transmission coefficient $T_t(\Delta)$ at the splice for a transverse mismatch d is found as (Hosain, Sharma and Ghatak, 1982; Gangopadhyay, Choudhury and Sarkar, 1999)

$$T_t(\Delta) = \left| \frac{C_t(\Delta)}{C_t(0)} \right|^2 \tag{2.8}$$

where, $\Delta = d/a$ and for the case $\Delta \leq 0.8$, the expression of $T_t(\Delta)$ is approximated as (Hosain, Sharma and Ghatak, 1982; Gangopadhyay, Choudhury and Sarkar, 1999)

$$\frac{C_t(\Delta)}{C_t(0)} = 1 - \frac{B_1}{B_0}\left(\frac{\Delta}{2}\right)^2 + \frac{B_2}{B_0}\left(\frac{\Delta}{2}\right)^4 - \frac{B_3}{B_0}\left(\frac{\Delta}{2}\right)^6 \tag{2.9}$$

where

$$B_0 = \int_0^\infty RdR \left| \cdot \ (R) \right|^2$$

$$B_1 = \int_0^\infty RdR \left| \frac{d\,\cdot}{dR} \right|^2$$

$$B_2 = \frac{1}{4}\left(\int_0^\infty RdR \left| \frac{d^2\,\cdot}{dR^2} \right|^2 + \int_0^\infty \frac{dR}{R} \left| \frac{d\,\cdot}{dR} \right|^2 \right) \tag{2.10}$$

$$B_3 = \frac{1}{36}\int_0^\infty RdR\left|\frac{d^3\cdot}{dR^3}\right|^2 - \frac{1}{12}\int_0^\infty \frac{dR}{R}\frac{d\cdot}{dR}\frac{d^3\cdot}{dR^3} - \frac{1}{24}\left(\frac{d^2\cdot}{dR^2}\right)_{R=0}$$

These integrals are evaluated by using the expression of ψ(R) given in Eq. (A13) together with the results (Watson, 1944; Gradshteyn and Ryzhik, 1980; Abramowitz and Stegun, 1981) given in Appendix B. Thus, we obtain (Gangopadhyay, Choudhury and Sarkar, 1999)

$$B_0 = 0.5[T_1 + T_2(T_3^2 - 1)]$$

$$B_1 = 4T_7 + 0.5T_2W_c^2[1 - T_3^2 + 2T_3/W_c] \tag{2.11}$$

$$B_2 = T_8 + [T_2W_c^4/8][T_3^2(1 + 2/W_c^2) - 1]$$

$$B_3 = T_9 + [T_2W_c^4/96][T_3^2(4 - W_c^2 + 20/W_c^2) + 4T_3(W_c + 6/W_c) + W_c^2 + 8]$$

where, T_1, T_2 and T_3 are given in Eq.(2.7) and T_7, T_8 and T_9 are found as

$$T_7 = A_2^2/4 + A_4^2/2 + 3A_6^2/4 + 2A_2A_4/3 + 3A_2A_6/4 + 6A_4A_6/5$$

$$T_8 = A_2^2 + 20A_4^2/3 + 117A_6^2/5 + 4A_2A_4 + 6A_2A_6 + 24A_4A_6 \tag{2.12}$$

$$T_9 = 2A_4^2 + 42.5A_6^2 + 18A_4A_6 - 2A_2A_4 - 5A_2A_6 - A_2/12$$

The transmission coefficient $T_a(p)$ for angular mismatch and the transmission coefficient $T_t(\Delta)$ for transverse mismatch at the splice are prescribed in terms of A_2, A_4 and A_6 and therefore, those can be predicted.

2.3 RESULTS AND DISCUSSIONS

In order to show the accuracy for our analytical formulations for $T_a(p)$ and $T_t(\Delta)$, we compare our results with the numerical exact results in case of both single-mode dispersion-shifted and dispersion-flattened fibers. In this context, we choose three typical trapezoidal fibers, each of V number 2.5, but of different aspect ratio (S) having values 0.25, 0.50 and 0.75 respectively (Paek, 1983) as example of dispersion-shifted fibers. Side by side, as regards dispersion-flattened fibers, we take two typical parabolic W fibers, each of V number 3.0, (Mishra, Hosain, Goyal and Sharma, 1984) and also two step W fibers each having value V number 2.0, (Monerie, 1982; Garth, 1989). Further, the parabolic W fibers taken here correspond to same C values but different ° values and so also is the case with step W fibers. In Fig. 2.1a, in case of three trapezoidal fibers taken for study, we plot the transmission coefficient $T_a(p)$ at the splice against normalised angular offset p for splicing of two identical trapezoidal fibers in each case. Similarly, in Fig. 2.1b, we present the variation of the transmission coefficient $T_t(\Delta)$ with normalised transverse offset Δ for splicing of the two identical trapezoidal fibers for three types of such fibers as chosen. Here, the results found by our formalism are indicated by markings while the exact numerical ones by solid lines. It is found that our results are indistinguishable from the exact numerical ones. Further, in Figs. 2.2a and 2.2b, we present the variation of the transmission coefficient $T_a(p)$ at the splice against normalised angular offset p and that of the transmission coefficient $T_t(\Delta)$ with normalised transverse offset Δ respectively in case of splicing of two identical parabolic W fibers for two typical fibers of this kind. As before, in Figs. 2.3a and 2.3b, we plot the transmission coefficient $T_a(p)$ at the splice against normalised angular offset p and the transmission coefficient $T_t(\Delta)$ against normalised transverse offset Δ respectively in case of splicing of two identical step W fibers for two types of such fibers. In Figs. 2.2a, 2.2b and 2.3a, 2.3b, we find that our results shown by distinct markings, as presented in the figures, match excellently with the exact numerical ones presented by solid lines. The figures show that the degree of tolerance at the splice for all kinds of fibers with respect angular mismatch is more than that with respect to transverse mismatch. In addition, for the typical step W fibers selected here, the degree of tolerance with respect to the said types of practical mismatches, is satisfactory. It is relevant to mention in this connection that increase of V number results in decrease of spot size causing more concentration near the core axis. Consequently, splice becomes less tolerant with respect to transverse offset. In fact, the present study will benefit the communication engineers from the point of view splicing of such important kinds of fibers.

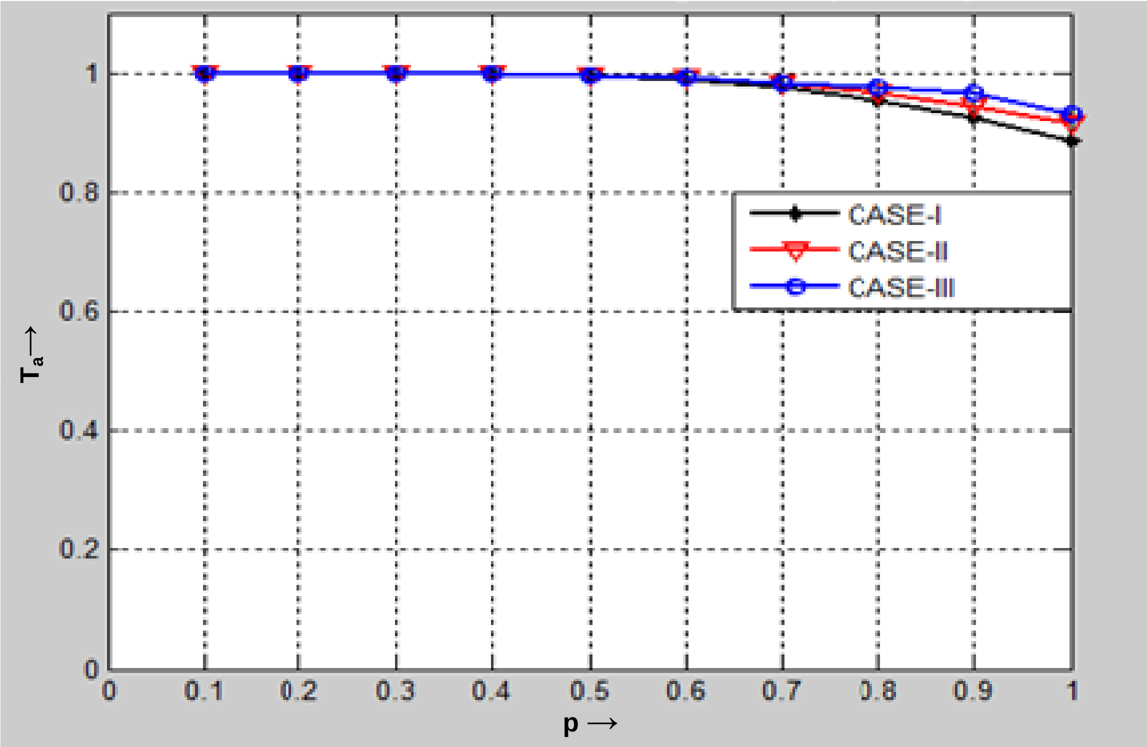

Fig. 2.1a: Power Transmission coefficient T_a versus normalised angular offset p for splicing of two identical Trapezoidal fibers in each case (V =2.5 for each fiber) (Case I – S=0.25), (Case II - S=0.50) and (Case III – S=0.75).

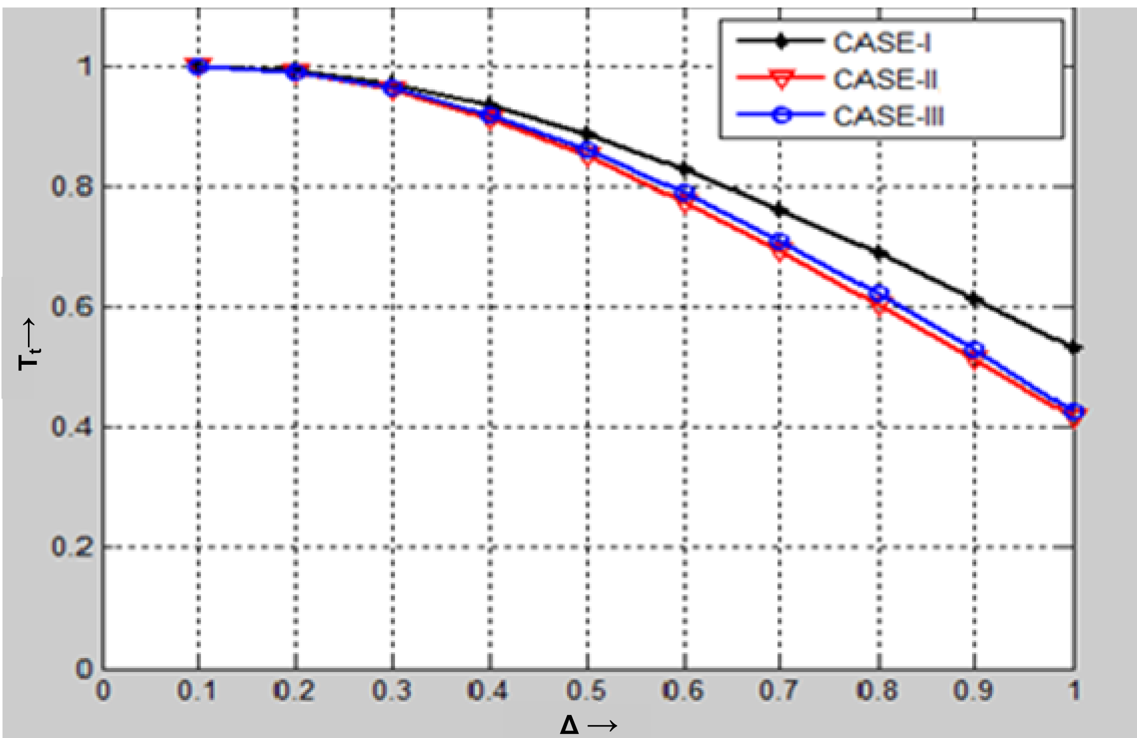

Fig. 2.1b: Power Transmission coefficient T_t versus normalised transverse offset Δ for splicing of two identical Trapezoidal fibers in each case (V=2.5 for each fiber) (Case I – S=0.25), (Case II - S=0.50) and (Case III – S=0.75).

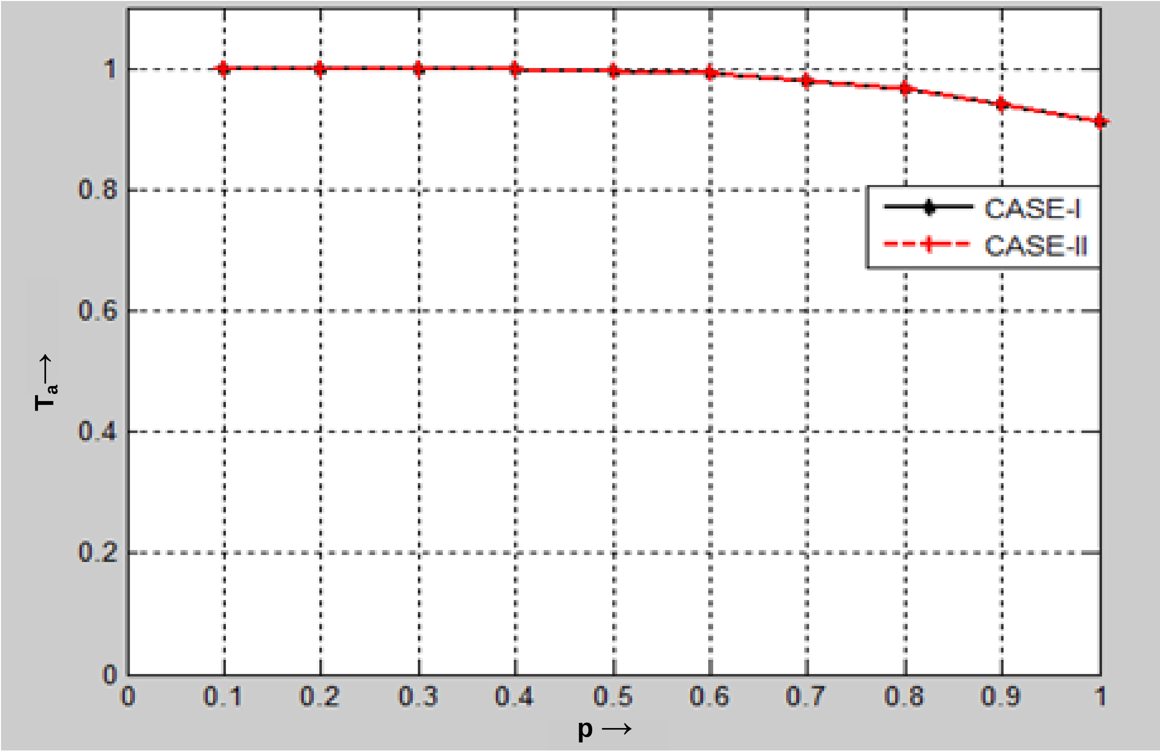

Fig. 2.2a: Power Transmission coefficient T_a versus normalised angular offset p for splicing of two identical parabolic W fibers in each case (V=3.0 for each fiber) (Case I - C=1.5 and ρ=1.4975) and (Case II - C=1.5 and ρ=1.5000).

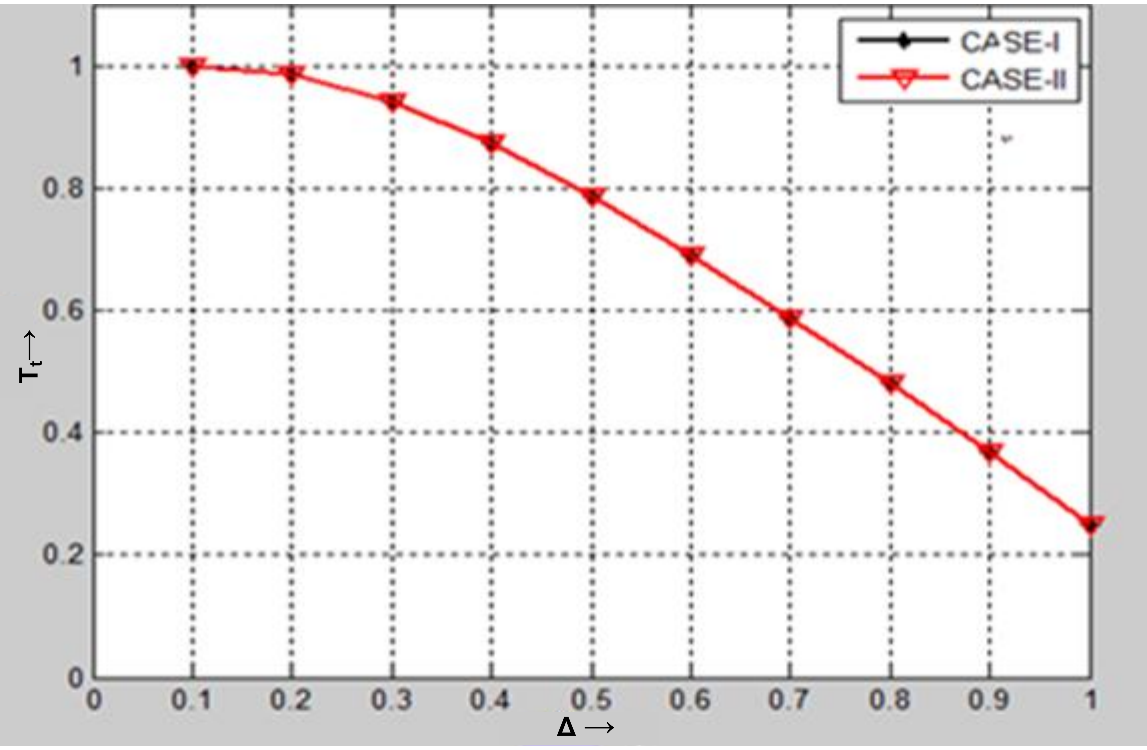

Fig. 2.2b: Power Transmission coefficient T_t versus normalised transverse offset Δ for splicing of two identical parabolic W fibers in each case (V=3.0 for each fiber) (Case I - C=1.5 and ρ=1.4975) and (Case II - C=1.5 and ρ=1.5000).

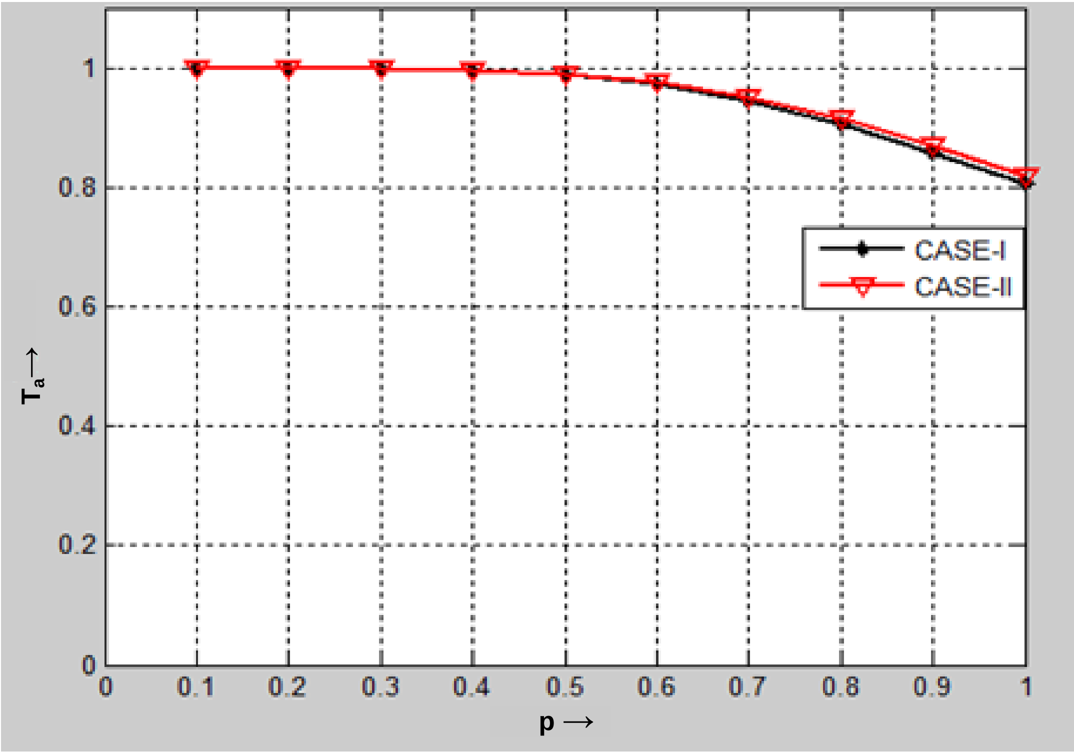

Fig. 2.3a: Power Transmission coefficient T_a versus normalised angular offset p for splicing of two identical step W fibers in each case (V=2.0 for each fiber) (Case I - C= 2 and ρ =1.3333) and (Case II - C=2 and ρ =1.2500).

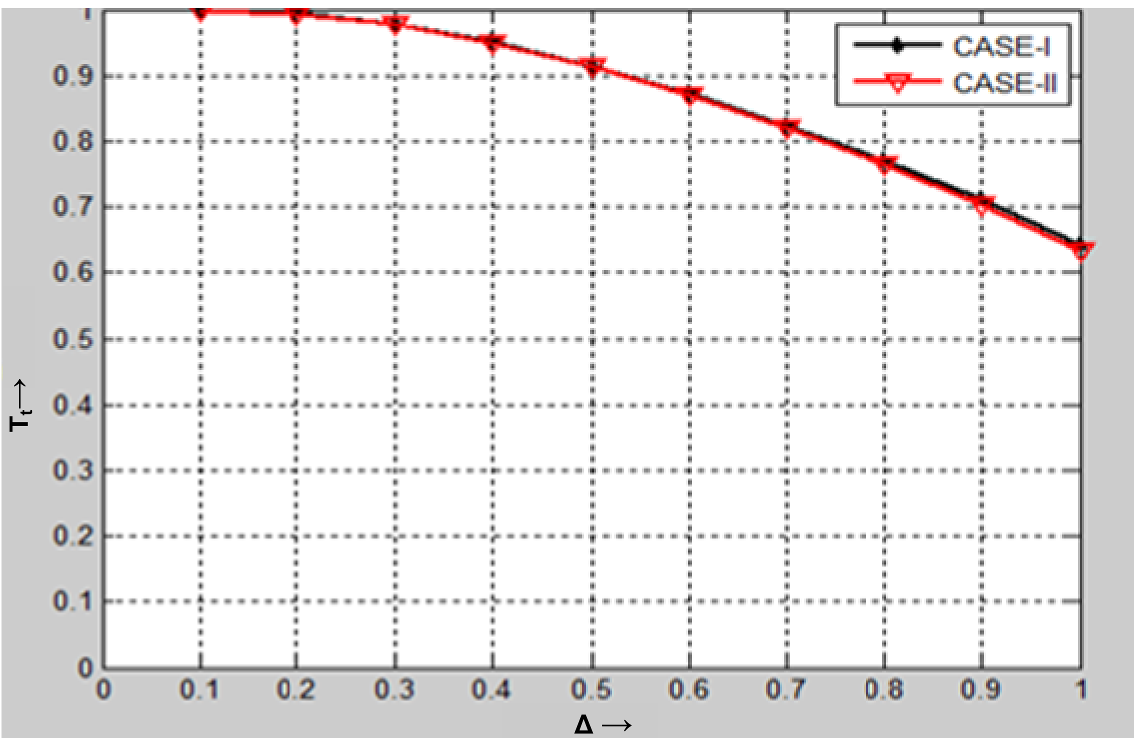

Fig. 2.3b: Power Transmission coefficient T_t versus normalised transverse offset Δ for splicing of two identical step W fibers in each case (V=2.0 for each fiber) (Case I - C= 2 and ρ =1.3333) and (Case II - C=2 and ρ =1.2500).

2.4 SUMMARY

Based on the simple power series expression of fundamental modal field in single-mode dispersion-shifted trapezoidal as well as dispersion-flattened graded and step W fibers, we formulate simple analytical formulations for power transmission coefficients at the splice separately for both angular and transverse mismatches. Further, splices being highly tolerant with respect to longitudinal separation, we restrict our investigations to the cases of transverse and angular mismatches only. Taking some typical trapezoidal fibers and parabolic as well as step W fibers as examples, we show that results found by our simple formalism agree excellently with the available exact numerical results. Further, the degree of tolerance at the splice for all kinds of fibers with respect angular mismatch is more than that with respect to transverse mismatch. The evaluations of the said parameters by this formalism require very little computations. Such excellent predictions leave scope for system engineers to use this user friendly but accurate formalism for study of other relevant characteristics concerned with all optical technology.

CHAPTER – 3

A NOVEL AND ACCURATE METHOD FOR ANALYSIS OF SINGLE-MODE DISPERSION-SHIFTED AND DISPERSION-FLATTENED FIBER DIRECTIONAL COUPLER

3.1 INTRODUCTION

Directional coupler formed by single-mode optical fibers using evanescent field coupling between two adjacent parallel single-mode fibers have come out as the most prospective device in optical-fiber sensor (Budiansky, Drucker, Kino and Rice, 1979; Hocker, 1979; Parriaux, Chartier and Bernoux, 1982; Nelson and Goss, 1982), nonlinear optics (Stegeman and Wright, 1990), wavelength filter (Digonnet and Shaw, 1983) etc. In order to design this type of coupling appropriately in various field of use, one should have proper knowledge on all types of coupling parameters. The directional coupling characteristics of step-index fiber have been accurately explained over the practical range of parameters (Eyges and Wintersteiner, 1981) using coupled-mode theory (Snyder and Love, 1983; Thyagarajan and Tewari, 1985). Based on Chebyshev formalism (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997), the characteristics of single-mode graded index fiber directional couplers (Sanyal, Gangopadhyay and Sarkar, 2000) have also been estimated accurately. However, to the best of our knowledge, there is no such analysis available on the characteristics of the directional couplers formed by a pair of dispersion-shifted trapezoidal fibers (Peak, 1983) or dispersion-flattened graded W fibers (Mishra, Hosain, Goyal and Sharma, 1984) or dispersion-flattened step W fibers (Monerie, 1982; Garth, 1989) till date. We know that, single-mode optical fiber emerges as the most effective medium in long-haul optical communication system. The operating wavelength in this context, ranges from 1.3 to 1.6 μm since silica has minimum attenuation loss to the extent of 0.15 dB/km at the wavelength 1.55 μm while material dispersion vanishes at the wavelength 1.3 μm. Thus, if zero dispersion wavelength is shifted to 1.55 μm, one can obtain minimum attenuation loss and zero dispersion simultaneously. Fibers, belonging to this category are called dispersion-shifted fibers and importance of those have been well realised in view of the fact that Erbium-doped fiber amplifier operates at wavelength 1.55 μm (Thyagarajan and Kakkar, 2004a; Pederson, 1994). In case of dispersion-flattened fibers, the material dispersion almost nullifies waveguide dispersion over a range of wavelengths. Accordingly such kind of fiber is best suited to enhance the information carrying capacity by means of wavelength division multiplexing (Olsson, Hegartz, Logen, Johnson, Walker, Cohen, Kasper and Campbell, 1985). Hence, they have a great importance in modern-day optical communication system. Therefore, it is very relevant to make a significant study on such type of fiber directional couplers. Moreover, investigations of different kinds of directional coupler are of tremendous importance in the field of optical technology. Accordingly, investigations performed in different laboratories have been constantly enriching the literature. In this respect, it is relevant to mention some research work as follows. The measurement of field bandwidth achievable with directional couplers using two dissimilar optical fibers is available in literature (Marcuse, 1985). Side by side, directional couplers with Kerr-type nonlinearity is also a potential problem in current scenario (Chaing, 1997). Several strategies for fabrication of porous sub-wavelength fibers together with measurement of transmission losses in terahertz using a novel non-destructive directional coupler method have also been reported (Dupuis, Allard, Morris, Stoeffler, Dubois and Skorobogatiy, 2009). Study of mismatched directional couplers has been added to literature (Snyder, Chen, Rowland and Mitchell, 1990). Taking into consideration that nonlinearity causes periodic mismatch between axially uniform twin cores, such study is also important in the context of nonlinear coupling (Snyder, Chen, Rowland and Mitchell, 1990). Thermo-optic modulation has been also employed for measurement of beat length in the directional coupler (Gnewuch, Román, Ulrich, Hempstead and Wilkinson, 1996). Further, photonic directional coupler has been designed as phase selector (Lee, Huang and Hsieh, 2013). Fabrication and measurement of a compact 3dB hybrid plasmonic directional coupler for silicon photonics integrated circuits have been successfully implemented (Caspers and Mojahedi, 2014). The investigations on substrate integrated waveguide based directional coupler for three dimensional integrated circuits have also been reported (Doghri, Djerafi, Ghiotto and Wu, 2015).

To use the coupled-mode theory, one should have clear idea about the modal field distributions of two single-mode fibers when they are non-interacting. The fundamental modal field for a single-mode dispersion-shifted and dispersion-flattened fiber can be calculated by applying numerical techniques or approximate methods. However, the simplest method under this circumstance is Gaussian variational method (Peng and Ankiewiez, 1991) but it does not predict the modal field distribution in the cladding region accurately. Again, the characteristic of single-mode parabolic core fiber directional coupler predicted by the Gaussian-exponential-Hankel function employing variational technique (Sharma, Hosain and Ghatak, 1982; Thyagarajan and Tewari, 1985) has also been reported with very high degree of accuracy. But this technique involves enormous computation. By using Chebyshev technique, a very simple but accurate power series expression has been reported for the fundamental mode of such dispersion-managed fibers (Bose, Gangopadhyay and Saha, 2012c). The simplicity as well as accuracy involved in the analysis of single-mode graded index fiber directional coupler by Chebyshev formalism (Sanyal, Gangopadhyay and Sarkar, 2000) has generated motivation for application of such reported formalism (Bose, Gangopadhyay and Saha, 2012c) for analysis of dispersion managed fiber directional couplers.

In this chapter, we report a simple but accurate analysis of coupling characteristics of directional coupler formed by either two identical single-mode dispersion-shifted trapezoidal fibers or dispersion-flattened graded W fibers or dispersion-flattened step W fibers. Here, our analysis is based on the simple series expression for the fundamental mode concerned with each kind of dispersion managed fiber.

3.2 THEORY

We consider directional coupler made of a pair of identical single-mode fibers having refractive index profile as expressed below,

$$n_s^2(R) = n_1^2\left[1 - \text{¢}\, f(R)\right], \qquad 0 < R \le 1$$

$$= n_2^2, \qquad R > 1 \tag{3.1}$$

where, $\text{¢} = \left(n_1^2 - n_2^2\right) / n_1^2$ is the grading parameter with n_1 and n_2 being the axial core and cladding refractive indices, with R being equal to r/a (a=core radius). Here, $f(R)$ defines the shape of the refractive index profile.

As illustrated in chapter 2, we present the profile function $f(R)$ relating to some dispersion-managed fibers for ready reference.

(I) Trapezoidal fiber (Paek, 1983)

$$
\begin{aligned}
f(R) &= 0 \ , & 0 < R \le S \\
f(R) &= \frac{R-S}{1-S} \ , & S < R \le 1
\end{aligned}
\tag{3.2}
$$

(II) Graded W fiber (Mishra, Hosain, Goyal and Sharma, 1984)

$$
\begin{aligned}
f(R) &= {}^\circ R^{q} \ , & R \le \frac{1}{C} \\
f(R) &= {}^\circ \ , & \frac{1}{C} < R \le 1
\end{aligned}
\tag{3.3}
$$

(III) Step W fiber (Monerie, 1982; Garth, 1989)

$$
\begin{aligned}
f(R) &= 0 \ , & R \le \frac{1}{C} \\
f(R) &= {}^\circ \ , & \frac{1}{C} < R \le 1
\end{aligned}
\tag{3.4}
$$

Here S stands for the aspect ratio for trapezoidal fiber and $1/C$ represents the normalised radial distance beyond which there occurs change of refractive index profile, as shown for Graded W and step W fibers. Further, q denotes the profile exponent in case of W fiber and its value is ∞ for step type profile. Here, $^\circ$ represents the relative index depth of inner cladding having refractive index n_i and it is expressed as,

$$
{}^\circ = \frac{n_1^2 - n_i^2}{n_1^2 - n_2^2} \ .
$$

The refractive index profiles of trapezoidal, graded W and step W fibers have been presented in Figs. 1.4, 1.5 and 1.6 respectively.

3.2.1 DISPERSION-SHIFTED TRAPEZOIDAL FIBER

Using Eq. (3.2) the refractive index profile for a directional coupler formed by a pair of such type of fibers can be written as (Thyagarajan and Tewari, 1985; Sanyal, Gangopadhyay and Sarkar, 2000),

$$n^2(R) = n_1^2, \qquad 0 < R_1 \le S \ \text{ and } \ 0 < R_2 \le S$$

$$= n_1^2\left[1 - \text{¢}\,\frac{R_1 - S}{1 - S}\right], \qquad S < R_1 \le 1$$

$$= n_1^2\left[1 - \text{¢}\,\frac{R_2 - S}{1 - S}\right], \qquad S < R_2 \le 1$$

$$= n_1^2\left[1 - \text{¢}\right] = n_2^2, \qquad \text{Otherwise} \tag{3.5}$$

where, $R_1 = r_1/a$ and $R_2 = r_2/a$ with r_1 and r_2 representing the radial distance from the centres of the two fibers to point P as shown in Fig. 3.1.

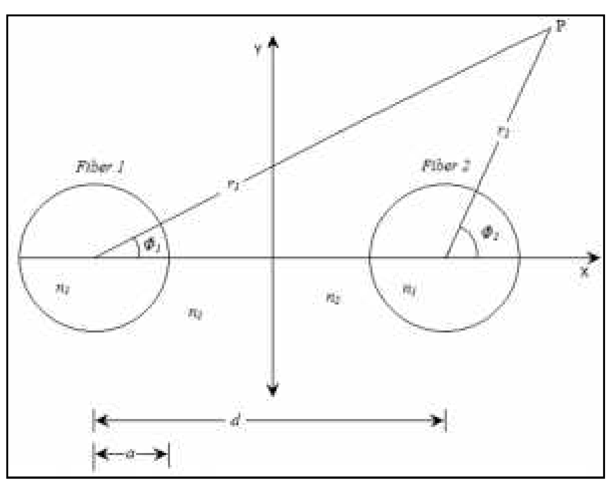

Fig. 3.1: Cross-section of a directional coupler formed by two identical single-mode optical fibers

Considering the coupled-mode theory under the condition of weak coupling, the coupling coefficient (¦) between the fibers can be written as (Snyder and Love, 1983; Thyagarajan and Tewari, 1985),

$$\kappa = \frac{k_0^2}{2\beta_1}\frac{\int_0^\infty\int_0^{2\pi}\left[n^2(R)-n_s^2(R)\right]\psi_1^*\psi_2 R_2\,dR_2\,d\theta_2}{\int_0^\infty\int_0^{2\pi}\psi_1^*\psi_1 R_1\,dR_1\,d\theta_1} \tag{3.6}$$

where, k_0 is free space wave number and β_1 is the propagation constant in the each fiber. ψ_1 and ψ_2 represent the fundamental modal field distribution for fiber 1 and fiber 2 respectively. In the above equation $\left[n^2(R)-n_s^2(R)\right]$ can expressed as,

$$\begin{aligned}\left[n^2(R)-n_s^2(R)\right] &= (n_1^2-n_2^2), && 0 < R_2 \le S\\ &= (n_1^2-n_2^2)\left(\frac{1-R_2}{1-S}\right), && S < R_2 \le 1\\ &= 0, && R_2 > 1\end{aligned} \tag{3.7}$$

As presented in Appendix A, we use the following linear relationship of $\frac{K_1(W)}{K_0(W)}$ with $\frac{1}{W}$, valid in the range $0.6 \le W \le 2.5$ (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997),

$$\frac{K_1(W)}{K_0(W)} = 1.034623 + 0.3890323/W \tag{3.8}$$

Again, as elucidated in Appendix A, the field inside the core and cladding is presented below (Bose, Gangopadhyay and Saha, 2012c)

$$\begin{aligned}\psi(R) &= a_0(1+A_2R^2+A_4R^4+A_6R^6), && R \le 1\\ &= a_0(1+A_2+A_4+A_6)\frac{K_0(W_cR)}{K_0(W_c)}, && R > 1\end{aligned} \tag{3.9}$$

where, $A_{2j} = \dfrac{a_{2j}}{a_0}$; $j = 1, 2, 3$ and W_c is the value of W as found by Chebyshev technique. The value of W and the constant A_{2j} for a given value of V can be easily calculated with little computations.

Using Eqs. (3.7), (3.8) and (3.9) in Eq. (3.6) one can obtain (Watson, 1944; Gradshteyn and Ryzhik, 1980; Abramowitz and Stegun, 1981) the expression of coupling coefficient $(¦)$ in case of dispersion-shifted trapezoidal fiber directional coupler as,

$$¦ = \frac{k_0^2\left(n_1^2 - n_2^2\right)}{{}_1}\frac{K_0\left(W_c \, d/a\right)}{K_0(W_c)}\frac{T_2\left[T_{27}I_1(W_cS) + T_{28}I_0(W_cS) + T_{29}I_1(W_c) + T_{30}I_0(W_c) + T_{31}\left\{\Psi(W_cS) - \Psi(W_c)\right\}\right]}{(1-S)\left[T_1 + T_2^2\left\{\dfrac{K_1^2(W_c)}{K_0^2(W_c)} - 1\right\}\right]} \quad (3.10)$$

The coupling length (L_c) which represents the minimum distance at which power is transferred from the input fiber to the other fiber, is given by,

$$L_c = \frac{®}{2¦} \quad (3.11)$$

In this context, it is necessary to mention that a small value of coupling length implies strong interaction. Using Eq. (3.10) and Eq. (3.11), we can express the normalised coupling length $\tilde{L}_c\left(= \dfrac{\sqrt{¢}\,L_c}{a}\right)$ as,

$$\tilde{L}_c = \frac{®}{V}\frac{K_0(W_c)}{K_0\left(W_c \, d/a\right)}\frac{(1-S)\left[\dfrac{T_1}{2} + \dfrac{T_2^2}{2}\left\{\dfrac{K_1^2(W_c)}{K_0^2(W_c)} - 1\right\}\right]}{T_2\left[T_{27}I_1(W_cS) + T_{28}I_0(W_cS) + T_{29}I_1(W_c) + T_{30}I_0(W_c) + T_{31}\left\{\Psi(W_cS) - \Psi(W_c)\right\}\right]} \quad (3.12)$$

where

$T_1 = 1 + A_2^2/3 + A_4^2/5 + A_6^2/7 + A_2 + 2A_4/3 + A_6/2 + A_2A_4/2 + 2A_2A_6/5 + A_4A_6/3$

$T_2 = 1 + A_2 + A_4 + A_6$

$T_3 = S^3/W_c + 4S/W_c^3$

$T_4 = 2S^2/W_c^2$

$T_5 = S^5/W_c + 16S^3/W_c^3 + 64S/W_c^5$

$T_6 = 4S^4/W_c^2 + 32S^2/W_c^4$

$T_7 = S^7/W_c + 36S^5/W_c^3 + 576S^3/W_c^5 + 2304S/W_c^7$

$T_8 = 6S^6/W_c^2 + 144S^4/W_c^4 + 1152S^2/W_c^6$

$T_9 = 1/W_c + 4/W_c^3$

$T_{10} = 2/W_c^2$

$T_{11} = 1/W_c + 16/W_c^3 + 64/W_c^5$

$T_{12} = 4/W_c^2 + 32/W_c^4$

$T_{13} = 1/W_c + 36/W_c^3 + 576/W_c^5 + 2304/W_c^7$

$T_{14} = 6/W_c^2 + 144/W_c^4 + 1152/W_c^6$

$T_{15} = 1/Wc + 9/W_c^3$

$T_{16} = 3/W_c^2$

$T_{17} = S^4/W_c + 9S^2/W_c^3$

$T_{18} = 3S^3/W_c^2$

$T_{19} = 1/W_c + 25/W_c^3 + 225/W_c^5$

$T_{20} = 5/W_c^3 + 75/W_c^4$

$T_{21} = S^6/W_c + 25S^4/W_c^3 + 225S^2/W_c^5$

$T_{22} = 5S^5/W_c^2 + 75S^3/W_c^4$

$T_{23} = 1/W_c + 49/W_c^3 + 1225/W_c^5 + 11025/W_c^7$

$$T_{24} = 7/W_c^2 + 245/W_c^4 + 3675/W_c^6$$

$$T_{25} = S^8/W_c + 49S^6/W_c^3 + 1225S^4/W_c^5 + 11025S^2/W_c^7$$

$$T_{26} = 7S^7/W_c^2 + 245S^5/W_c^4 + 3675S^3/W_c^6$$

$$T_{27} = \left(T_{17} - ST_3\right)A_2 + \left(T_{21} - ST_5\right)A_4 + \left(T_{25} - ST_7\right)A_6$$

$$T_{28} = \left(ST_4 - T_{18}\right)A_2 + \left(ST_6 - T_{22}\right)A_4 + \left(ST_8 - T_{26}\right)A_6$$

$$T_{29} = \left(T_9 - T_{15}\right)A_2 + \left(T_{11} - T_{19}\right)A_4 + \left(T_{13} - T_{23}\right)A_6$$

$$T_{30} = \left(T_{16} - T_{10}\right)A_2 + \left(T_{20} - T_{12}\right)A_4 + \left(T_{24} - T_{14}\right)A_6$$

$$T_{31} = 1/W_c^3 + 9A_2/W_c^5 + 225A_4/W_c^7 + 11025A_6/W_c^9$$

3.2.2 DISPERSION-FLATTENED GRADED W FIBER

Now, we consider a directional coupler is formed by two identical dispersion-flattened graded W fibers. Therefore, the refractive index profile will be (using Eq. (3.3)) (Thyagarajan and Tewari, 1985; Sanyal, Gangopadhyay and Sarkar, 2000),

$$n^2(R) = n_1^2\left(1-\text{¢}^{\circ} R_1^q\right), \qquad 0 < R_1 \le 1/C$$

$$= n_1^2\left(1-\text{¢}^{\circ} R_2^q\right), \qquad 0 < R_2 \le 1/C$$

$$= n_1^2\left(1-\text{¢}^{\circ}\right), \qquad 1/C < R_1 \le 1 \text{ and } 1/C < R_2 \le 1$$

$$= n_1^2\left(1-\text{¢}\right) = n_2^2, \qquad \text{otherwise.} \tag{3.13}$$

Hence, for this type of couplers the term $\left[n^2(R) - n_s^2(R)\right]$ will be,

$$\left[n^2(R)-n_s^2(R)\right]=\left(n_1^2-n_2^2\right)\left(1-\circ R_2^q\right), \qquad 0<R_2\le 1/C$$

$$=\left(n_1^2-n_2^2\right)\left(1-\circ\right), \qquad 1/C<R_2\le 1 \tag{3.14}$$

$$=0, \qquad R_2>1$$

where, q represents the profile exponent of the fiber and for graded W fiber, we consider parabolic profile and accordingly, we take q=2. Therefore, the coupling coefficient (¦) of dispersion-flattened graded W fiber directional coupler is obtained (Watson, 1944; Gradshteyn and Ryzhik, 1980; Abramowitz and Stegun, 1981**)** by using Eqs. (3.8), (3.9) and (3.14) in Eq. (3.6),

$$\text{¦}=\frac{k_0^2\left(n_1^2-n_2^2\right)}{1}\frac{K_0\left(W_c\, d/a\right)}{K_0\left(W_c\right)}\frac{T_2\left[\left(1-\circ\right)\left\{T_{40}I_1\left(W_c\right)-T_{41}I_0\left(W_c\right)\right\}-\circ\left\{T_{42}I_1\left(W_c/C\right)-T_{43}I_0\left(W_c/C\right)\right\}\right]}{\left[T_1+T_2^2\left\{\frac{K_1^2(W_c)}{K_0^2(W_c)}-1\right\}\right]} \tag{3.15}$$

Corresponding normalised coupling length $\left(\tilde{L}_c\right)$ will be,

$$\tilde{L}_c=\frac{\text{®}}{V}\frac{K_0(W_c)}{K_0\left(W_c\, d/a\right)}\frac{\left[\frac{T_1}{2}+\frac{T_2^2}{2}\left\{\frac{K_1^2(W_c)}{K_0^2(W_c)}-1\right\}\right]}{T_2\left[\left(1-\circ\right)\left\{T_{40}I_1\left(W_c\right)-T_{41}I_0\left(W_c\right)\right\}-\circ\left\{T_{42}I_1\left(W_c/C\right)-T_{43}I_0\left(W_c/C\right)\right\}\right]} \tag{3.16}$$

where

$$T_{32}=1/W_cC^3+4/W_c^3C$$

$$T_{33}=2/W_c^2C^2$$

$$T_{34}=1/W_cC^5+16/W_c^3C^3+64/W_c^5C$$

$$T_{35}=4/W_c^2C^4+32/W_c^4C^2$$

$$T_{36}=1/W_cC^7+36/W_c^3C^5+576/W_c^5C^3+2304/W_c^7C$$

$$T_{37}=6/W_c^2C^6+144/W_c^4C^4+1152/W_c^6C^2$$

$$T_{38} = 1/W_c C^9 + 64/W_c^3 C^7 + 2304/W_c^5 C^5 + 36864/W_c^7 C^3 + 147456/W_c^9 C$$

$$T_{39} = 8/W_c^2 C^8 + 384/W_c^4 C^6 + 9216/W_c^6 C^4 + 73728/W_c^8 C^2$$

$$T_{40} = 1/W_c + A_2 T_9 + A_4 T_{11} + A_6 T_{13}$$

$$T_{41} = A_2 T_{10} + A_4 T_{12} + A_6 T_{14}$$

$$T_{42} = \left(T_{32} - 1/W_c C\right) + A_2\left(T_{34} - T_{32}\right) + A_4\left(T_{36} - T_{34}\right) + A_6\left(T_{38} - T_{36}\right)$$

$$T_{43} = T_{33} + A_2\left(T_{35} - T_{33}\right) + A_4\left(T_{37} - T_{35}\right) + A_6\left(T_{39} - T_{37}\right)$$

3.2.3 DISPERSION-FLATTENED STEP W FIBER

In case of coupler formed by a pair of identical dispersion-flattened step W fibers, the refractive-index profile corresponding to coupled structure is given by (Thyagarajan and Tewari, 1985; Sanyal, Gangopadhyay and Sarkar, 2000),

$$\begin{aligned} n^2(R) &= n_1^2, && 0 < R_1 \le 1/C \text{ and } 0 < R_2 \le 1/C \\ &= n_1^2\left(1 - \text{¢°}\right), && 1/C < R_1 \le 1 \text{ and } 1/C < R_2 \le 1 \\ &= n_1^2\left(1 - \text{¢}\right) = n_2^2, && \text{otherwise.} \end{aligned} \quad (3.17)$$

Thus, the expression of $\left[n^2(R) - n_s^2(R)\right]$ will be,

$$\begin{aligned} \left[n^2(R) - n_s^2(R)\right] &= \left(n_1^2 - n_2^2\right), && 0 < R_2 \le 1/C \\ &= \left(n_1^2 - n_2^2\right)\left(1 - \text{°}\right), && 1/C < R_2 \le 1 \\ &= 0, && R_2 > 1 \end{aligned} \quad (3.18)$$

Further, substituting Eqs. (3.8), (3.9) and (3.18) in Eq. (3.6), we find (Watson, 1944; Gradshteyn and Ryzhik, 1980; Abramowitz and Stegun, 1981) the expression of coupling coefficient (¦) for dispersion-flattened step W fiber directional coupler as below,

$$\text{¦} = \frac{k_0^2\left(n_1^2-n_2^2\right)}{{}_1}\frac{K_0\left(W_c\, d/a\right)}{K_0\left(W_c\right)}\frac{T_2\left[(1-\text{°})\left\{T_{40}I_1\left(W_c\right)-T_{41}I_0\left(W_c\right)\right\}+\text{°}\left\{T_{44}I_1\left(W_c/C\right)-T_{45}I_0\left(W_c/C\right)\right\}\right]}{\left[T_1+T_2^2\left\{\dfrac{K_1^2(W_c)}{K_0^2(W_c)}-1\right\}\right]} \tag{3.19}$$

Accordingly, in case of dispersion-flattened step W fiber directional coupler, the normalised coupling length can be expressed as,

$$\tilde{L}_c = \frac{\text{®}}{V}\frac{K_0(W_c)}{K_0\left(W_c\, d/a\right)}\frac{\left[\dfrac{T_1}{2}+\dfrac{T_2^2}{2}\left\{\dfrac{K_1^2(W_c)}{K_0^2(W_c)}-1\right\}\right]}{T_2\left[(1-\text{°})\left\{T_{40}I_1\left(W_c\right)-T_{41}I_0\left(W_c\right)\right\}+\text{°}\left\{T_{44}I_1\left(W_c/C\right)-T_{45}I_0\left(W_c/C\right)\right\}\right]} \tag{3.20}$$

where

$T_{44} = 1/W_cC + A_2T_{32} + A_4T_{34} + A_6T_{36}$

$T_{45} = A_2T_{33} + A_4T_{35} + A_6T_{37}$.

3.3 RESULTS AND DISCUSSIONS

Using our formulation, we have evaluated normalised coupling length and plotted it against normalised centre-to-centre core separation D(=d/a) of two identical fibers. The study is conducted for some typical dispersion-managed fibers. Here, the solid line in Fig. 3.2 shows the variation of normalised coupling length ($\tilde{L}_c$) with normalised separation (D) for a directional coupler comprising two identical dispersion-shifted trapezoidal fibers, each having V value 2.5 and aspect ratio (S) equal to 0.5.Similarly, the broken line in Fig. 3.2 represents $\tilde{L}_c$ versus D for directional coupler consisting of two identical trapezoidal fibers, each having V and S values as 2.5 and 0.75 respectively. Such choice of D and S values has been borrowed from Ref. (Paek, 1983). Further, it is pertinent to mention that first higher order mode cut-off V values for trapezoidal fibers with S= 0.50 and 0.75 are respectively 3.1979 and 2.7801 (Paek, 1983).

Again, in Fig. 3.3, we have plotted the normalised coupling length $(\tilde{L}_c)$ versus the normalised separation (D) by solid line for a directional coupler consisting of a pair of identical dispersion-flattened graded W fibers, each having V number 3.0, C value 1.5 and ρ value 1.4975. Here, the broken line in Fig. 3.3, presents the variation of $\tilde{L}_c$ with D for the directional coupler containing two identical dispersion-flattened graded W fibers, each being characterised by V=3.0, C value 1.5 and ρ=1.5000 . Further, in case of choice of C and ρ values for graded W fiber, Ref. (Mishra, Hosain, Goyal and Sharma, 1984) has been followed. Side by side, it can be pointed out that first higher mode cut-off V values for graded W fibers having ρ=1.4975, C=1.5 and ρ=1.5, C=1.5 are respectively 4.8452 and 4.8490 (Mishra, Hosain, Goyal and Sharma, 1984).

In Fig. 3.4, the solid curve shows the variation of the normalised coupling length with normalised separation for directional coupler made of two identical dispersion-flattened step W fibers, each with V=2.0, 1/C = 0.5 and ρ=1.3333. Again, the change of $\tilde{L}_c$ with D for a directional coupler comprising two identical dispersion-flattened step W fibers, each of V, 1/C and ρ values as 2.0, 0.5 and 1.2500 respectively, is represented by broken line in Fig. 3.4. It is relevant to mention in this context that Ref. (Garth, 1989) has been used to select step W fiber having 1/C = 0.5 and ρ=1.3333 while the step W fiber characterized by 1/C = 0.5 and ρ=1.2500 has been chosen following reference (Monerie, 1982). Further, the first higher order mode cut off V values corresponding to the said step W fibers having ρ=1.3333 and ρ=1.2500 are respectively 2.7789 (Garth, 1989) and 2.8000 (Monerie, 1982).

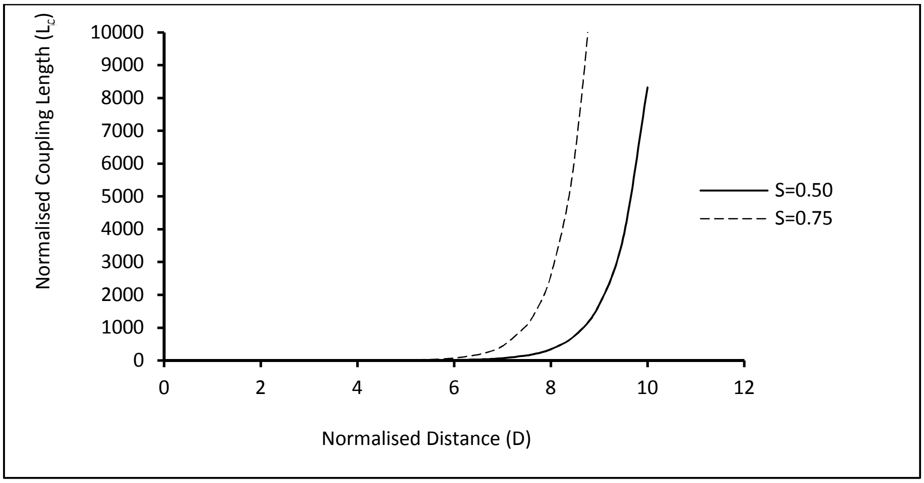

Fig. 3.2: Variation of normalised coupling length $\widetilde{L_c}$ with normalised separation D of directional coupler consisting of two dispersion-shifted trapezoidal fibers (V=2.5) for different aspect ratio (S) values

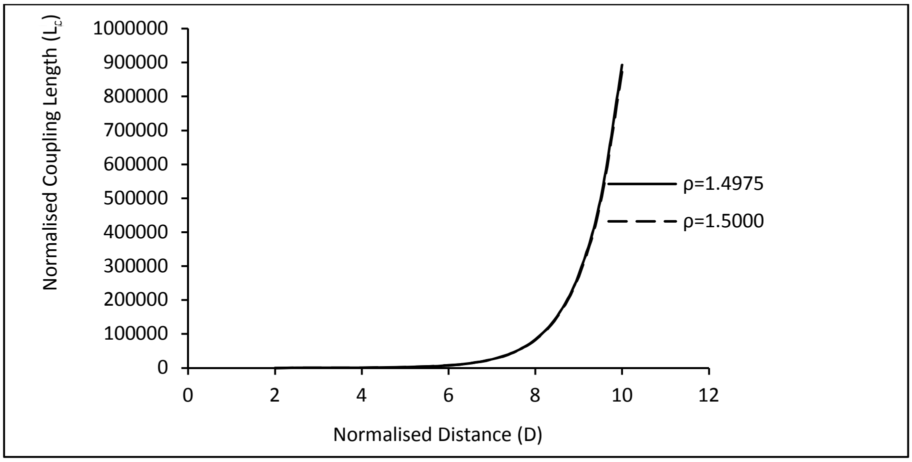

Fig. 3.3: Variation of normalised coupling length $\widetilde{L_c}$ with normalised separation D of directional coupler consisting of two dispersion-flattened graded W fibers (V=3.0) for different ρ values

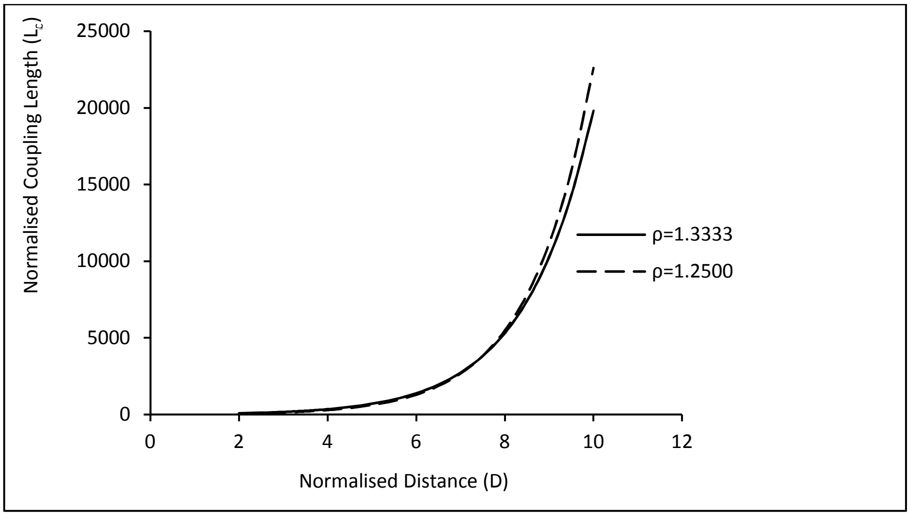

Fig. 3.4: Variation of normalised coupling length $\tilde{L_c}$ with normalised separation D of directional coupler consisting of two dispersion-flattened step W fibers (V=2.0) for different ρ values

It deserves mentioning in this connection that a directional coupler containing two identical graded index single-mode fibers, each having core radius a=3 µm, δ=0.004 and normalised coupling length ($\tilde{L}_c$) less than 200 (Thyagarajan and Tewari, 1985) will imply that corresponding coupling length (L_c) must be less than 9.5 mm. The results found have close agreement with those predicted in case of directional coupler comprising two identical grade index fibers (Thyagarajan and Tewari, 1985). In this context, the model proposed for a step index fiber in the form of an equivalent slab guiding structure justifies the results obtained (Sharma, Kompella and Mishra, 1990). Moreover, the analysis reported in reference (Sanyal, Gangopadhyay and Sarkar, 2000) also verifies the accuracy of the results obtained. Accordingly, our study will focus on the choice of D value for which the coupling length remains less than 9.5 mm in case of each directional coupler under investigation. We tabulate below the respective D values for different directional couplers used, where coupling length remains less than 9.5 mm. In Table 3.1, the relevant values have been presented and side by side, the corresponding graphs also indicate the same.

TABLE 3.1

Normalised Distance(D) Values for Directional Couplers of Different Type of Optical Fibers.

Trapezoidal fiber (Figure 3.2)		**Graded W fiber (Figure 3.3)**		**Step W fiber (Figure 3.4)**	
Aspect ratio(S)	**Normalised Distance(D)**	**Relative index depth (ρ)**	**Normalised Distance(D)**	**Relative index depth (ρ)**	**Normalised Distance(D)**
0.5	7.65	1.4975	3	1.25	3.5
0.75	6.55	1.5	3.05	1.3333	3.25

3.4 SUMMARY

Using the coupled-mode theory and Chebyshev power series expression for fundamental mode of dispersion managed fiber, we prescribe analytical formulation of normalised coupling length in terms of fiber to fiber separation for a directional coupler containing two identical single-mode dispersion managed fibers. The said estimations have been made for directional couplers corresponding to some typical dispersion managed fibers and the results obtained are comparable to available numerical results in case of directional coupler formed of two identical single-mode graded index fibers. The concerned calculations require simple and little computation and as such it will prove user friendly to technologists who are working in the field of optical technology.

CHAPTER – 4

PREDICTION OF FUNDAMENTAL MODAL FIELD FOR GRADED INDEX FIBER IN PRESENCE OF KERR NONLINEARITY

4.1 INTRODUCTION

Investigation of propagation characteristics of single-mode optical fiber in the nonlinear region (Tomlinson, Stolen and Chank, 1984; Tai, Tomita, Jewell and Hasegawa, 1986; Snyder, Chen, Poladian and Mitchel, 1990; Goncharenko, 1990; Sammut and Pask, 1990; Agrawal and Boyd, 1992) is of great interest since the intensity of optical beam changes the refractive index profile of those fibers. As a result, the fibers subjected to high optical intensity show propagation characteristics different from those relating to the same fibers operating in the linear region. There are different kinds of nonlinearity like third order, fifth order etc. and saturable non linearity as well (Saitoh, Fujisawa, Kirihara and Koshiba, 2006). Generation of a particular kind of nonlinearity will be dependent on the intensity of optical beam and also on the doped medium of the fiber (Agrawal, 2013). The presence of nonlinearity leads to compression of pulse while dispersion causes broadening of the pulse and thus interplay between dispersion and nonlinearity results in propagation of optical beam as such (Agrawal and Boyd, 1992). This is known as propagation of optical soliton, the study of which happens to be an emerging matter in the field of optical communication system. Kerr–type nonlinearity imposes constraint on the performance of optical fiber communication. However, cancellation of signal to signal interaction can be obtained by superposition of twin waves at the end of transmission line (Liu, Chraplyvy, Winzer, Tkach and Chandrasekhar, 2013). The study of nonlinear interference noise generated in space division multiplexed transmission through optical fiber is also an emerging problem (Antonelli, Golani, Shtaif and Mecozzil, 2017).

Further, the strong Kerr-type nonlinearity clubbed with high quality microresonator provides the necessary platform for integrated nonlinear photonics (Lu, Lee, Rogers and Lin, 2014). The influence of Kerr-type nonlinearity in opto-mechanical ring resonator has also been demonstrated (Yu, Ren, Zhang, Bourouina, Tan, Tsai and Liu, 2012). Estimation of propagation characteristics along with field distribution of single-mode fiber as well as photonic crystal fibers subjected to Kerr effect has already been reported (Seaton, Valera, Shoemaker, Stegeman, Chilwell and Smith, 1985; Hayata, Koshiba and Suzuki, 1987; Okamoto and Marcatili, 1989; Khijwania, Nair and Sarkar, 2009; Diaz-Soriano, Ortiz-Mora and Dengra, 2013). But the numerical methods involved in this context require lengthy computation. Hence, a simple but accurate study of effect of optical Kerr-type nonlinearity on the said kinds of fibers in single-mode region is in demand in literature. Based on Chebyshev technique, a simple power series expression of fundamental mode of graded index fiber has been formulated (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997; Patra, Gangopadhyay and Sarkar, 2000). This formulation of fundamental mode has predicted the propagation parameters of single-mode graded index fibers efficiently (Gangopadhyay and Sarkar, 1997a; Gangopadhyay and Sarkar, 1998a; Gangopadhyay and Sarkar, 1998b; Patra, Gangopadhyay and Sarkar, 2001a; Patra, Gangopadhyay and Sarkar, 2001b) over a wide range of normalised frequency (V) values. This method involves formulation of linear relationship of $\frac{K_1(W)}{K_0(W)}$ with $\frac{1}{W}$ by applying least square fitting technique. Here, W represents the cladding decay parameter. Applying this series expression for fundamental modal field, splice losses for step, parabolic and triangular index fibers in presence of transverse and angular mismatches have also been evaluated (Gangopadhyay, Choudhury and Sarkar, 1999; Debnath and Gangopadhyay, 2016). Further, it has been found that the said predictions, though evaluated in a simple fashion, agree excellently well with those found on the basis of analytical expression for step index fiber and Gaussian Hankel-exponential approximation for parabolic index fiber (Hosain, Sharma and Ghatak, 1982).

It is relevant to mention in this connection that application of Chevbyshev technique for evaluation of first higher order mode cut-off frequency in case of optical fibers having Kerr-type nonlinearity, has produced excellent results (Roy and Sarkar, 2016). Moreover, Chebyshev formalism has also been successfully employed to analyse the propagation constants of single-mode optical fibers having Kerr-type nonlinearity (Sadhu, Karak and Sarkar, 2013). Thus, recalling that the evaluation of modal field is an important issue in predicting various propagation parameters associated with Kerr-type nonlinear fibers, we, in this chapter, report estimation of modal field for some typical step and parabolic index fibers. In this context, we have applied iterative technique. Further, we have compared our results with the available exact results found by finite element method involving variational technique (Hayata, Koshiba and Suzuki, 1987).

4.2 THEORY

As described in chapters 2 and 3, we, for the sake of ready reference, present the refractive index profile $n(R)$ for a weakly guiding circular core optical fiber as below

$$n^2(R) = \begin{cases} n_1^2(1-2\Delta\ f(R)), & R \le 1 \\ n_2^2\ , & R > 1 \end{cases} \tag{4.1}$$

where, $R = r/a$, a= core radius and $\Delta = (n_1^2 - n_2^2)/2n_1^2$ with n_1 and n_2 being the refractive indices of the core axis and the cladding respectively. Here, $f(R)$ represents the shape of refractive index profile and in case of graded index fiber, it is given by,

$$f(R) = R^q \tag{4.2}$$

where, q is the profile exponent whose values for step and parabolic index fibers are ∞ and 2 respectively.

The refractive index $n(R)$ of the fiber involving Kerr-type nonlinearity is denoted as (Mondal and Sarkar, 1996)

$$n^2(R) = n_L^2(R) + \frac{n_2^2 n_{NL}(R)}{\eta_0}\psi^2(R) \tag{4.3}$$

where, $\eta_0 (= (\mu_0/\varepsilon_0)^{1/2})$ with μ_0 and ε_0 presenting respectively the permeability and permittivity of free space and $n_L(R)$, $n_{NL}(R)$ denoting the linear refractive index and the distribution of nonlinear Kerr coefficient (m^2/W) respectively. The modal field of circularly symmetric scalar nonlinear LP_{01} mode is given as $\psi(R)$ which obeys the following scalar wave equation (Mondal and Sarkar, 1996)

$$\frac{d^2 \cdot (R)}{dR^2} + \frac{1}{R}\frac{d \cdot (R)}{dR} + [V^2(1 - f(R)) - W^2] \cdot (R) + V^2 g(R) \ \cdot\ ^3(R) = 0 \qquad (4.4)$$

together with the boundary condition at core cladding interface as

$$\left[\frac{1}{\cdot}\frac{d\cdot}{dR}\right]_{R=1} = -\left[\frac{WK_1(W)}{K_0(W)}\right] \qquad (4.5)$$

where W is the cladding decay parameter and

$$g(R) = \frac{n_2 n_{NL} P}{® \, a^2 (n_1^2 - n_2^2)}$$

where K_1 and K_0 are the modified Bessel functions of first and zero order respectively.

The field in the cladding is, however, given by

$$\cdot (R) \sim K_0(WR) \quad \text{for} \quad R>1$$

where, $V\left[= k_0 a(n_1^2 - n_2^2)^{1/2}\right]$ is the normalised frequency while $W\left[= a(\ ^2 - n_2^2 k_0^2)^{1/2}\right]$ is the cladding delay parameter. Here, and k_0 are the propagation constant and free space wave number respectively.

Following (Shijun, 1987; Chen, 1982), the power series expression for the fundamental modal field in graded index fiber based on Chebyshev formalism is approximated as (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997; Patra, Gangopadhyay and Sarkar, 2000)

$$\begin{aligned} \cdot (R) &= (a_0 + a_2 R^2 + a_4 R^4 + a_6 R^6), && R \le 1 \\ &= (a_0 + a_2 + a_4 + a_6)\frac{K_0(WR)}{K_0(W)}, && R > 1 \end{aligned} \qquad (4.6)$$

where, a_0, a_2, a_4, a_6 are constants.

The Chebyshev points are given by (Chen, 1982)

$$R_m = \cos\left(\frac{2m-1}{2M-1}\frac{®}{2}\right), \qquad m = 1, 2, \ldots, (M-1) \tag{4.7}$$

By putting M=4 appropriate for Eq. (4.7), we get three values of R as follows

R_1=0.9749, R_2=0.7818 and R_3=0.4338 (4.8)

Now applying Eq.(4.6) in Eq.(4.4), we get

$$\begin{aligned} &a_0[V^2(1-f(R_i))-W^2+V^2 g\cdot\ ^2(R_i)]+a_2[4+R_i^2\{V^2(1-f(R_i))-W^2+V^2 g\cdot\ ^2(R_i)\}] \\ &+a_4[16R_i^2+R_i^4\{V^2(1-f(R_i))-W^2+V^2 g\cdot\ ^2(R_i)\}] \\ &+a_6[36R_i^4+R_i^6\{V^2(1-f(R_i))-W^2+V^2 g\cdot\ ^2(R_i)\}]=0 \end{aligned} \tag{4.9}$$

where i = 1, 2, 3.

Application of least square fitting in the interval $0.6 \le W \le 2.5$ leads to the following expression (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997)

$$\frac{K_1(W)}{K_0(W)} = \sim + \frac{\quad}{W} \tag{4.10}$$

where α= 1.034623 and β= 0.3890323.

However, when W is less than 0.6, linear least square fitting technique is applied in order to make linear formulation of $\frac{K_1(W)}{K_0(W)}$ as a function of $\frac{1}{W}$ for different short intervals of W (Patra, Gangopadhyay and Sarkar, 2000). Accordingly α and β for different short intervals will be different.

Using Eqs. (4.6) and (4.10) in Eq. (4.5), we get

$$a_0(\sim W + \quad) + a_2(\sim W + 2 + \quad) + a_4(\sim W + 4 + \quad) + a_6(\sim W + 6 + \quad) = 0 \tag{4.11}$$

The nontrivial solution of a_2, a_4, a_6 in terms a_0 from three equations in (4.9) and one equation in (4.11) requires the following condition

$$\begin{vmatrix} \sim_1 & {}_1 & \text{¥}_1 & \text{¢}_1 \\ \sim_2 & {}_2 & \text{¥}_2 & \text{¢}_2 \\ \sim_3 & {}_3 & \text{¥}_3 & \text{¢}_3 \\ \sim_4 & {}_4 & \text{¥}_4 & \text{¢}_4 \end{vmatrix} = 0 \tag{4.12}$$

here,

$$\begin{aligned} \sim_i &= V^2(1 - f(R_i)) - W^2 + V^2 g \cdot {}^2(R_i) \\ {}_i &= 4 + R_i^2[(V^2(1 - f(R_i)) - W^2) + V^2 g \cdot {}^2(R_i)] \\ \text{¥}_i &= 16R_i^2 + R_i^4[(V^2(1 - f(R_i)) - W^2) + V^2 g \cdot {}^2(R_i)] \\ \text{¢}_i &= 36R_i^4 + R_i^6[(V^2(1 - f(R_i)) - W^2) + V^2 g \cdot {}^2(R_i)] \end{aligned} \tag{4.13}$$

where i = 1, 2, 3 and

$$\begin{aligned} \sim_4 &= \sim W + \\ {}_4 &= 2 + \sim_4; \\ \text{¥}_4 &= 4 + \sim_4; \\ \text{¢}_4 &= 6 + \sim_4; \end{aligned}$$

The solution of Eq. (4.12) appears complicated due to the presence of the terms like $\cdot\ {}^2(R_i)$. Therefore, we first solve Eq. (4.12) for the linear region by taking g=0 whereby one can get W value for a particular V. Next, this W value and corresponding V value are used in any three of four equations given by Eqs. (4.9) and (4.11) to obtain a_2, a_4 and a_6 in terms of a_0 in the linear domain. Then, a particular value of g is taken and we resort to iteration technique a number of times until for a particular V value in case of a particular kind of fiber, convergent values of W and a_2, a_4 and a_6 in terms of a_0 are found.

4.3 RESULTS AND DISCUSSIONS

Here, we estimate cladding decay parameter in presence of Kerr-type nonlinearity for different V values in case of step and parabolic index single-mode fibers and we compare our results with the exact results for the fundamental mode (Hayata, Koshiba and Suzuki, 1987). Here, the values taken for refractive index of cladding (n_2) and basic fiber parameter $a\sqrt{\left(n_1^2 - n_2^2\right)}$ are 1.47 and 0.22 μm respectively (Mondal and Sarkar, 1996; Roy and Sarkar, 2016). Further, it deserves mentioning in this connection that Eq. (4.3) shows that nonlinear Kerr coefficient $n_{NL}(R)$ takes care of intensity dependent part of refractive index as a multiplicative constant only at a particular value of R. On the other hand, the change of refractive index at a particular value of R due to incident light intensity, can be evaluated only when both nonlinear Kerr coefficient $n_{NL}(R)$ (m^2/W) and power P (W) are considered simultaneously. This necessitates introduction of the nonlinear term $n_{NL}(R)$P (m^2) for the present study. Here, the range of $n_{NL}P$ varies from -1.5×10^{-14} m^2 to 1.5×10^{-14} m^2 (Mondal and Sarkar, 1996; Roy and Sarkar, 2016). Further, self-focusing and self-defocussing effects correspond to positive and negative values of $n_{NL}P$ respectively.

In Fig. 4.1, we present the variation of cladding decay parameter with Kerr-type nonlinearity parameter for three typical single-mode step index fibers having V numbers 1.4, 2.0 and 2.4. Similarly, in Fig. 4.2, we represent the graph of cladding decay parameter versus Kerr-type nonlinearity parameter in case of three typical parabolic index fibers having V numbers 2.5, 3.0 and 3.5.

In Figs. 4.3, 4.4 and 4.5, we plot variation of fundamental modal field with respect to normalised radial distance in presence and in absence of Kerr-type nonlinearity for step index fibers having V numbers 1.4, 2.0 and 2.4 (close to first higher order cut-off) respectively. Similarly, in Figs. 4.6, 4.7 and 4.8, we show the variation of fundamental modal field with respect to normalised radial distance in presence and in absence of Kerr-type nonlinearity for parabolic index fibers having V numbers 2.5, 3.0 and 3.5 (close to first higher order cut-off) respectively. Here, we observe that the modal field increases with the decrease in the V number in case of both step and parabolic index fibers. Further, it is seen that with increase of V number in case of both kinds of fibers, the effect of nonlinearity in predominant. For example, in case of step index fiber having V=1.4, the field near the core axis in presence of nonlinearity is slightly separated from that in absence of nonlinearity, while for step index fiber having V=2.4, the modal field near the core axis is influenced largely by nonlinearity as seen from Fig. 4.5. Such behavior is also applicable for parabolic index fiber. This is consistent with the prediction that with increase in V number, the effective area decreases resulting in more nonlinear effect (Majumdar, Das and Gangopadhyay, 2014). The influence of nonlinearity within the entire core along with some portion of cladding have been presented in the figures for the sake of presentation of response to such nonlinear fibers. Moreover, in domains where nonlinearity is exploited (Yu, Ren, Zhang, Bourouina, Tan, Tsai and Liu, 2012), the results found will lead to suitable selection of V number. Here, in Figs. 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, we have presented our predictions by + corresponding to $n_{NL}P = 1.5 \times 10^{-14}$, by * corresponding to $n_{NL}P = -1.5 \times 10^{-14}$ and by ○ corresponding to $n_{NL}P=0$. Further in all the said figures, solid lines like ——— correspond to the exact values obtained by finite element based variational method (Hayata, Koshiba and Suzuki, 1987). We find that out results are virtually indistinguishable from the exact ones. Further, taking $n_{NL}P=0$, we predict the fundamental modal field for fiber in absence of nonlinearity and here the results match excellently with available exact results (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997). Thus, one may reliably predict the fundamental modal field for single-mode graded index Kerr-type nonlinear fiber by solving a (4x4) determinant by this iterative Chebyshev formalism. Thus, one should choose V number judiciously so that performance of communication is not reduced owing to predominance of nonlinearity.

Conclusively, the effect of nonlinearity is investigated in such kinds of fibers, the operations of which are being powered by several watts. It is worth saying that the results found can be utilised in reducing modal noise in concerned devices.

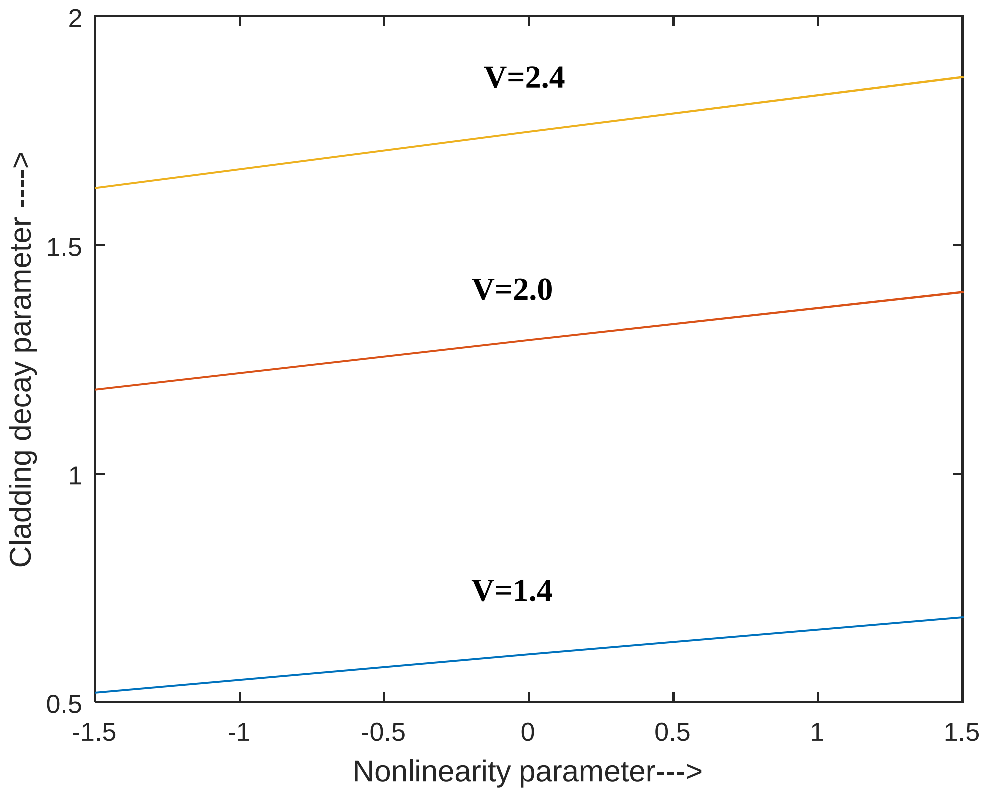

Fig. 4.1: Variation of Cladding decay parameter for fundamental mode against nonlinearity parameter $n_{NL}P$ ($x10^{-14}$ m^2) in case of step index fibers having different V numbers.

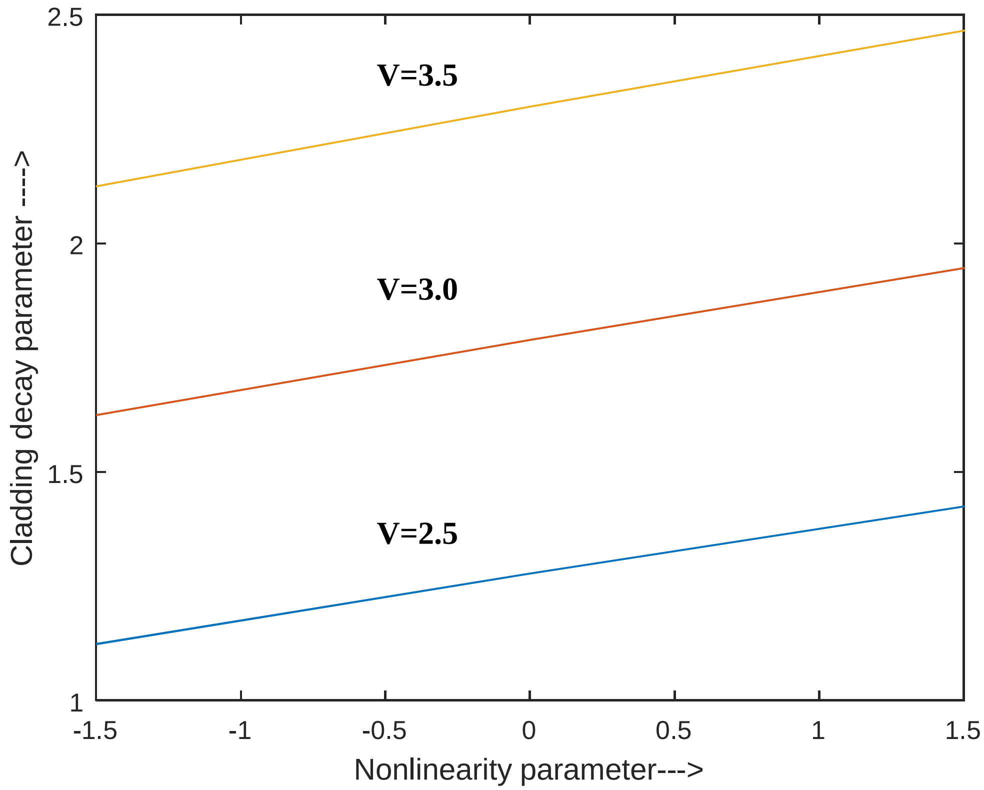

Fig. 4.2: Variation of Cladding decay parameter for fundamental mode against nonlinearity parameter $n_{NL}P$ (x10^{-14} m^2) in case of parabolic index fibers having different V numbers.

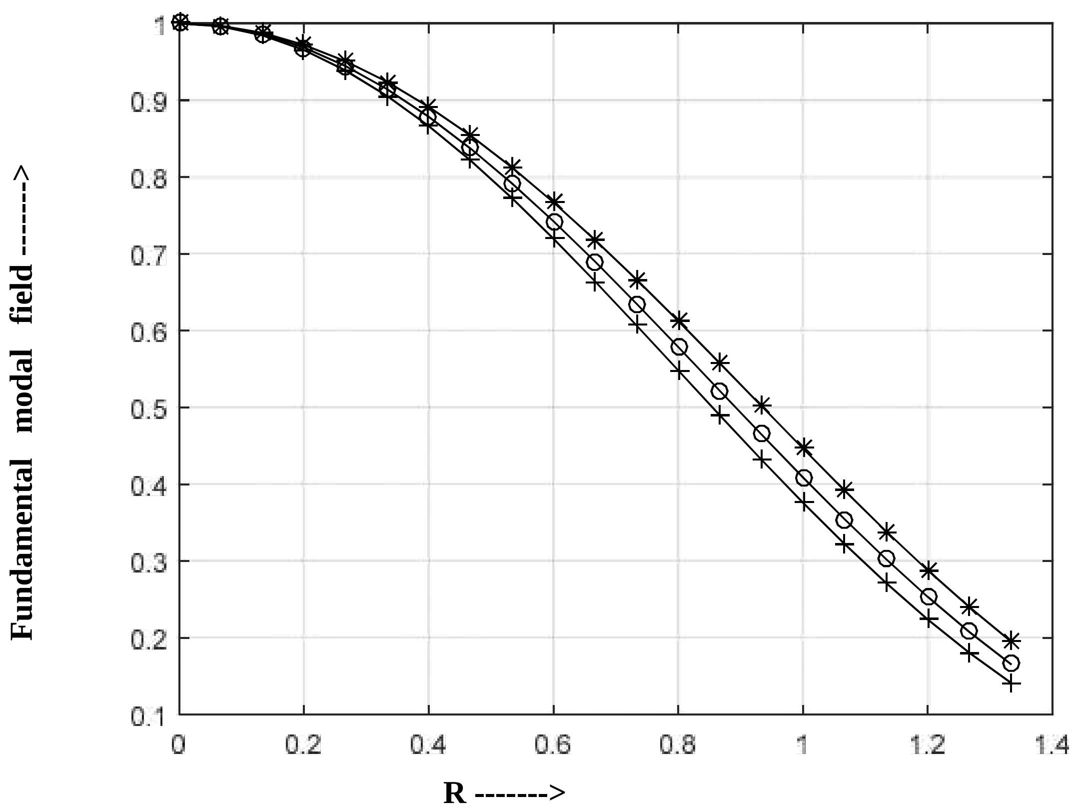

Fig. 4.3: Variation of fundamental modal field Ψ(R) against normalised radial distance R in case of step index fiber having V=1.4 for different nonlinearity parameter $n_{NL}P$ (+ for $n_{NL}P = 1.5x10^{-14}$, * for $n_{NL}P = -1.5x10^{-14}$, O for $n_{NL}P = 0$: Our results and ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987) .

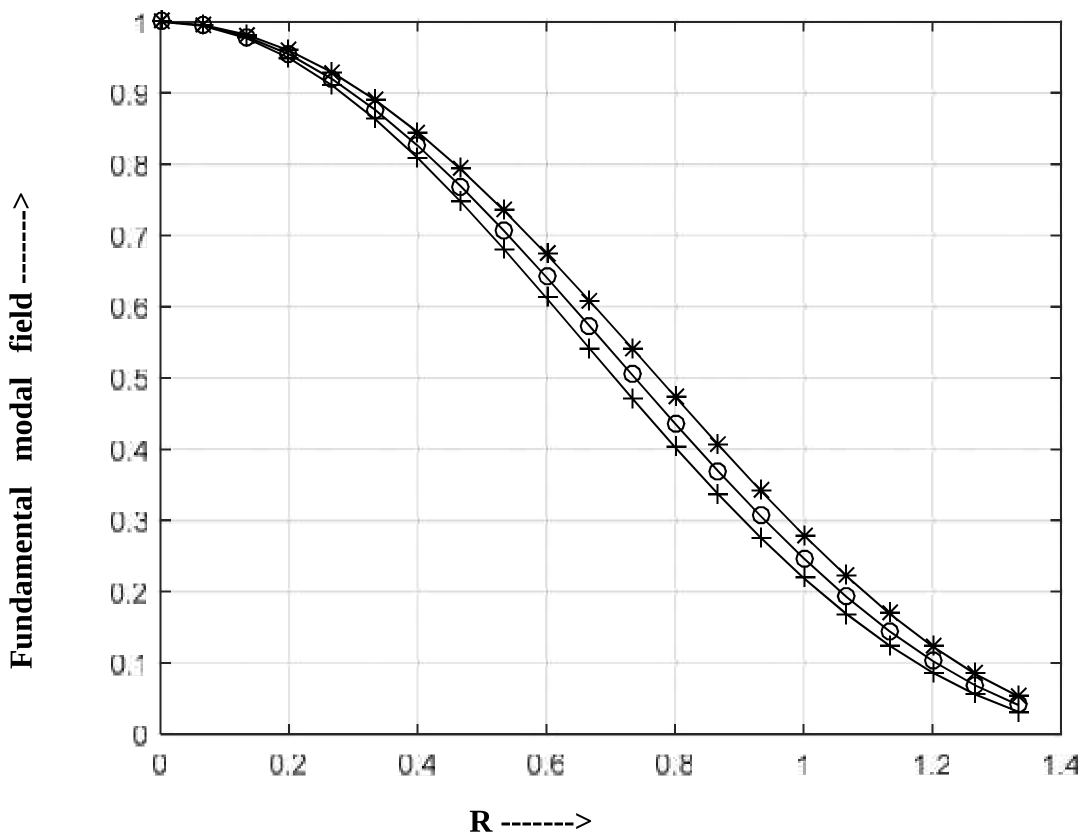

Fig. 4.4: Variation of fundamental modal field Ψ(R) against normalised radial distance R in case of step index fiber having V=2.0 for different nonlinearity parameter $n_{NL}P$ (+ for $n_{NL}P = 1.5x10^{-14}$, * for $n_{NL}P = -1.5x10^{-14}$, O for $n_{NL}P = 0$: Our results and ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

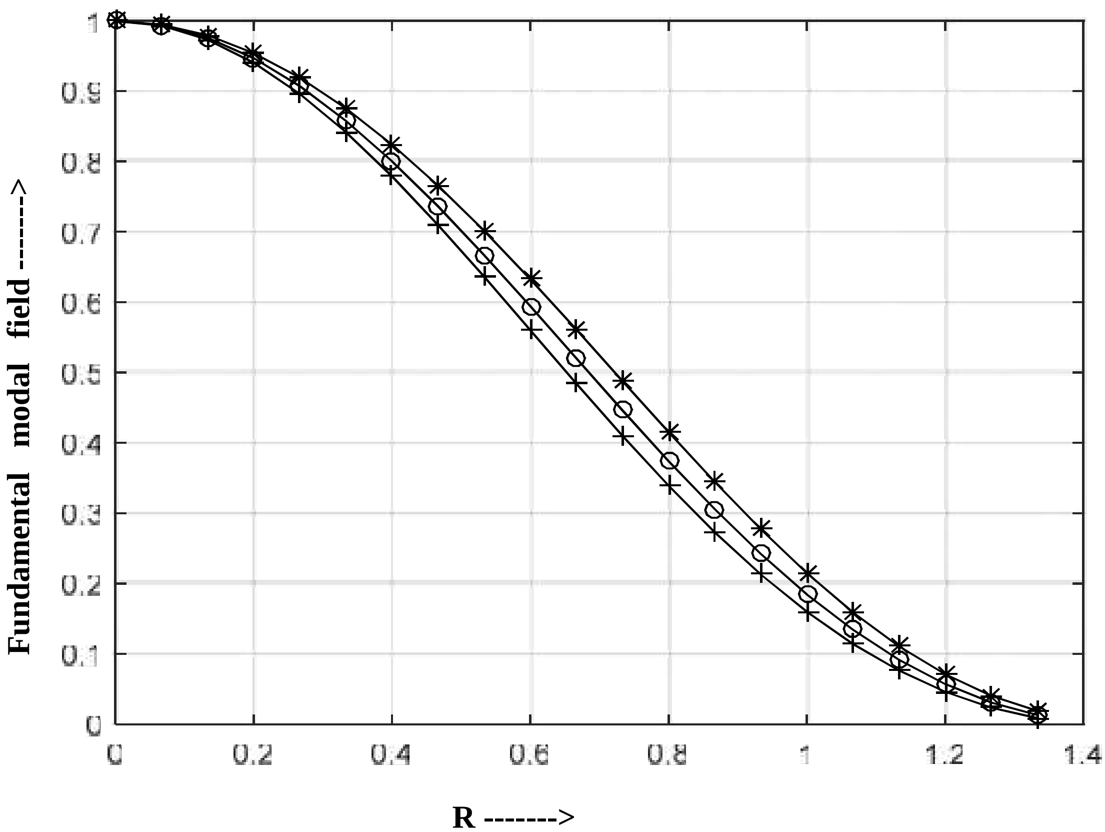

Fig. 4.5: Variation of fundamental modal field Ψ(R) against normalised radial distance R in case of step index fiber having V=2.4 for different nonlinearity parameter $n_{NL}P$ (+ for $n_{NL}P = 1.5\text{x}10^{-14}$, * for $n_{NL}P = -1.5\text{x}10^{-14}$, O for $n_{NL}P = 0$: Our results and ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

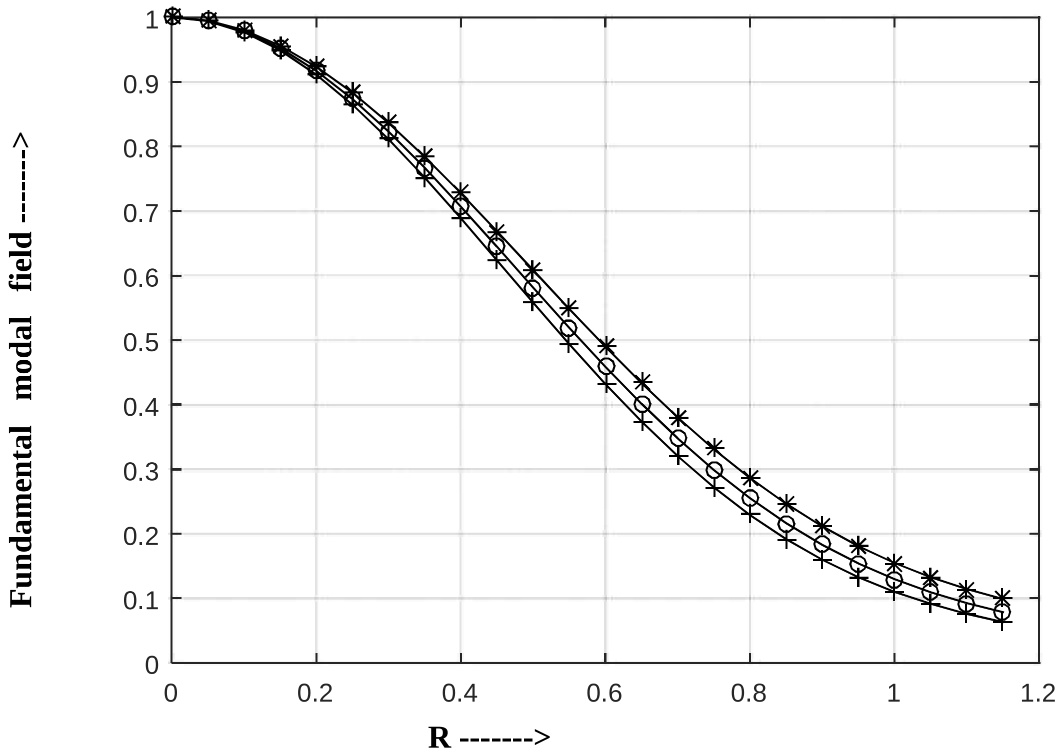

Fig. 4.6: Variation of fundamental modal field Ψ(R) against normalised radial distance R in case of parabolic index fiber having V=2.5 for different nonlinearity parameter $n_{NL}P$ (+ for $n_{NL}P = 1.5x10^{-14}$, * for $n_{NL}P = -1.5x10^{-14}$, O for $n_{NL}P = 0$: Our results and ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

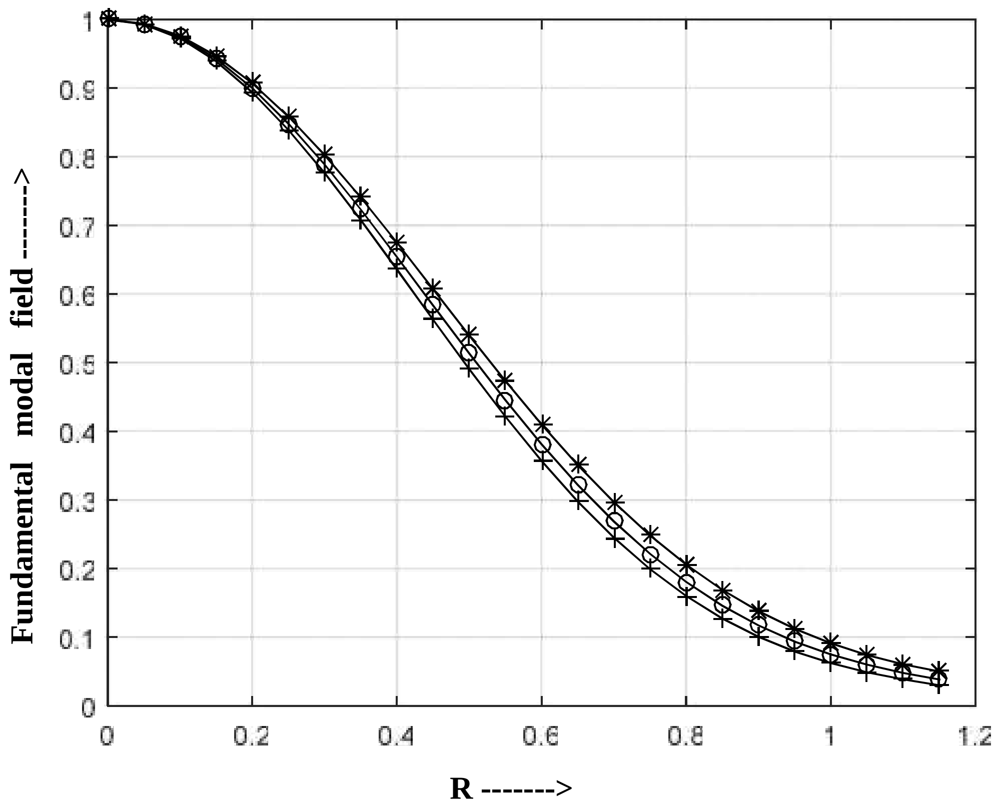

Fig. 4.7: Variation of fundamental modal field Ψ(R) against normalised radial distance R in case of parabolic index fiber having V=3.0 for different nonlinearity parameter (+ for $n_{NL}P = 1.5 \times 10^{-14}$, * for $n_{NL}P = -1.5 \times 10^{-14}$, O for $n_{NL}P = 0$: Our results and ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

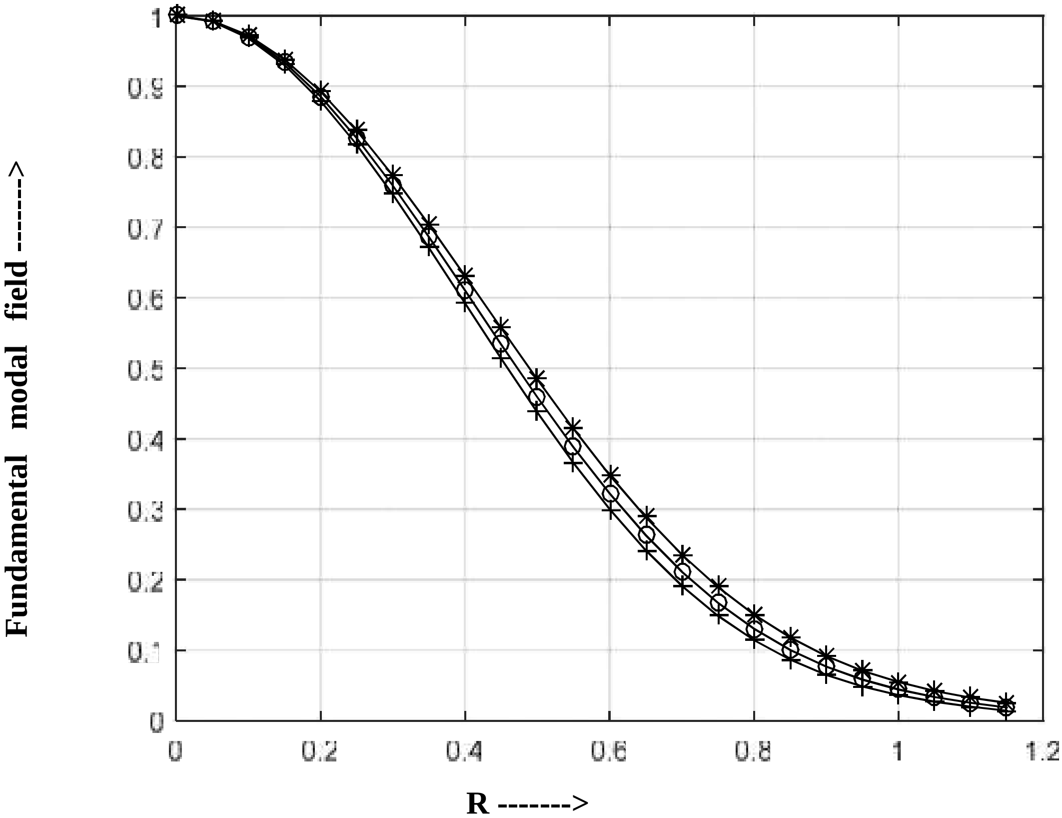

Fig. 4.8: Variation of fundamental modal field Ψ(R) against normalised radial distance R in case of parabolic index fiber having V=3.5 for different nonlinearity parameter (+ for $n_{NL}P = 1.5x10^{-14}$, * for $n_{NL}P = -1.5x10^{-14}$, O for $n_{NL}P = 0$: Our results and ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

4.4 SUMMARY

We have prescribed a simple, novel but accurate method based on iteration in order to predict the fundamental modal field of single-mode graded index fiber in presence of Kerr-type nonlinearity. Our formalism is based on the power series formulation for the modal field of single-mode graded index fibers by Chebyshev formalism. The results found can be used for minimisation of modal noise in the field of optical communication. Further, the results may prove extremely important for different kinds of sensors and in the field of integrated nonlinear photonics. This method opens up a simple but accurate technique for estimation of various propagation characteristics of Kerr-type nonlinear graded index fibers. The accuracy of this simple formalism leaves scope for extension of the analysis for the study of other kinds of fibers.

CHAPTER – 5

PREDICTION OF FIRST HIGHER ORDER MODAL FIELD FOR GRADED INDEX FIBER IN PRESENCE OF KERR NONLINEARITY

5.1 INTRODUCTION

Prediction of propagation parameters of optical fiber in presence of nonlinearity (Tomlinson, Stolen and Chank, 1984; Tai, Tomita, Jewell and Hasegawa, 1986; Snyder, Chen, Poladian and Mitchel, 1990; Goncharenko, 1990; Sammut and Pask, 1992; Agrawal and Boyd, 1992) is a potential problem since the intensity of optical beam modifies the refractive index profile of those fibers. Thus, nonlinear fibers exposed to optical beam show propagation parameters different from those in the linear region.

Nonlinearity can be categorised as third order, fifth order etc. including saturable nonlinearity (Saitoh, Fujisawa, Kirihara, and Koshiba, 2006). Intensity of optical medium as well as the nature of doping decide which kind of nonlinearity will come into action (Agrawal, 2013). The presence of nonlinearity causes pulse compression while dispersion broadens the pulse and accordingly dispersion and nonlinearity acting together causes propagation of optical beam as such (Agrawal and Boyd, 1992). This is what is called propagation of optical soliton which is a matter of huge interest in terms of investigation in the present fiber optic communication system. The presence of Kerr–type nonlinearity influences the performance of optical fiber communication. However, signal to signal interaction can be cancelled by superposition of twin waves at the end of transmission line (Liu, Chraplyvy, Winzer, Tkach and Chandrasekhar, 2013). Further, the study of nonlinear interference noise produced in space division multiplexed transmission through optical fiber has emerged as a potential problem (Antonelli, Golani, Shtaif and Mecozzi1, 2017). Moreover, the strong Kerr-type nonlinearity in presence of high quality micro resonator leads to the platform for integrated nonlinear photonics (Lu, Lee, Rogers and Lin, 2014). The role of Kerr-type nonlinearity in opto-mechanical ring resonator has also been reported (Yu, Ren, Zhang, Bourouina, Tan, Tsai and Liu, 2012).

Further, dual-mode optical fiber has also emerged as an important medium in the context of optical fiber communication system (Spajer and Charquille, 1986; Eguchi, 2001; Eguchi, Koshiba and Tsuji, 2002). It has been found that the first higher order mode of dual-mode optical fiber has large negative waveguide dispersion and this can be used to cancel the positive dispersion. Thus, such a dispersion compensating dual-mode optical fiber can be designed to operate around 1.55 μm, the wavelength at which erbium-doped-fiber amplifier usually operates (Pedersen, 1994). Further, double-layer profile core dispersion-shifted fiber has lower bending and transmission losses compared to simple core-cladding dispersion-shifted fibers (Monerie, 1982). Therefore, a dual-mode fiber with a double-layer profile emerges as a suitable optical device in respect of broad band transmission of communication at wavelength 1.55µm on account of its low bending and transmission losses and dispersion compensation as well. Again, methods have been prescribed for separate estimation of LP_{01} and LP_{11} mode losses in dual-mode optical fiber (Ohashi, Kitayama, Kobayashi and Ishida, 1984). The spatial technique involving fundamental and first higher order mode has been successfully used for estimation of time difference relating to group delay concerned with the two modes (Shibata, Tateda, Seikai and Uchida, 1980). Further, in the field of sensor technology, group delay between fundamental and first higher mode is being successfully employed for the associated needful (Bohnert and Pequignot, 1998).

Finite element solution relating to graded index slab waveguides in presence of nonlinearity is available in literature (Hayata, Koshiba and Suzuki, 1987). Moreover, the numerical methods involved for such investigations require lengthy computation. In this context, a simple but accurate study on effect of optical Kerr nonlinearity on first higher order mode cut-off frequency for the different kinds of fibers are in demand in literature. Further, formalism regarding evaluation of first higher order cut-off frequency of nonlinear optical fibers has been reported (Mondal and Sarkar, 1996; Roy and Sarkar, 2016). The formalism (Roy and Sarkar, 2016) is based on Chebyshev method and the results found are accurate, though the concerned execution is simple. In addition, literature has already been enriched in respect of report of method involving Chebyshev technique for prediction of propagation parameters in single-mode nonlinear fibers (Sadhu, Karak and Sarkar, 2013). Chebyshev power series technique (Chen, 1982; Shijun, 1987) applied for estimation of propagation parameters of linear fibers has shown the features of simplicity combined with accuracy as well (Gangopadhyay and Sarkar, 1998b; Patra, Gangopadhyay and Goswami, 2008; Bose, Gangopadhyay and Saha, 2012c). Being motivated by the effectiveness of Chebyshev formalism in the domain of linear fibers, we have recently reported application of this formalism for accurate prediction of fundamental modal field for graded index fibers in presence of Kerr-type nonlinearity. This has been presented in chapter 4.

Taking into consideration that evaluation of first higher order modal field associated with Kerr-type nonlinearity is also as important matter in the study of dual-mode fiber, we in this chapter, present application of Chebyshev formalism (Patra, Gangopadhyay and Goswami, 2008) to estimate the first higher order modal field for some typical step and graded index fibers. Here, we have applied iterative technique involving Chebyshev formalism in order to predict the first higher order modal field. Further, we have shown excellent match between our results and the exact results which can be obtained by finite element method (Hayata, Koshiba and Suzuki, 1987).

5.2 THEORY

As presented in chapters 2, 3 and 4, we express the refractive index profile $n(R)$ for a weakly guiding circular core fiber as follows

$$n^2(R) = \begin{cases} n_1^2(1 - 2\Delta f(R)), & R \leq 1 \\ n_2^2\ , & R > 1 \end{cases} \qquad (5.1)$$

where $R = r/a$ and $\Delta = (n_1^2 - n_2^2)/2n_1^2$ with a being the core radius and n_1, n_2 being the refractive indices of the core axis and the cladding respectively. Here, $f(R)$ represents the shape of refractive index profile of the fiber.

The profile functions $f(R)$ for different kinds of fibers are given below,

(I) $f(R) = 0,\qquad 0 < R \leq 1\qquad$ for step index fiber

(5.2)

(II) $f(R) = R^2,\qquad 0 < R \leq 1\qquad$ for parabolic index fiber

The refractive index of the fiber involving Kerr-type nonlinearity is expressed as (Mondal and Sarkar, 1996)

$$n^2(R) = n_L^2(R) + \frac{n_2^2 n_{NL}(R)}{\eta_0} \psi^2(R) \tag{5.3}$$

where $\eta_0 = (\mu_0/\varepsilon_0)^{1/2})$ with μ_0 and ε_0 presenting respectively the permeability and permittivity of free space and $n_{NL}(R)$ denoting the distribution of nonlinear Kerr coefficient (m^2/W). The modal field of circularly symmetric scalar nonlinear LP_{11} mode is given as $\psi(R)$ which satisfies the following scalar wave equation (Mondal and Sarkar, 1996)

$$\frac{d^2\psi(R)}{dR^2} + \frac{1}{R}\frac{d\psi(R)}{dR} + [V^2(1 - f(R)) - W^2]\psi(R) - \frac{\psi(R)}{R^2} + V^2 g(R)\psi^3(R) = 0 \tag{5.4}$$

where

$g(R) = \dfrac{n_2 n_{NL} P}{\text{®}\, a^2(n_1^2 - n_2^2)}$ along with the following boundary condition

$$\left[\frac{1}{\psi}\frac{d\psi}{dR}\right]_{R=1} = -\left[1 + \frac{WK_0(W)}{K_1(W)}\right] \tag{5.5}$$

where, K_1, K_0 are the modified Bessel functions of first and zero order respectively while $V\left[= k_0 a(n_1^2 - n_2^2)^{1/2}\right]$ and $W\left[= a(\ {}^2 - n_2^2 k_0^2)^{1/2}\right]$ are the normalised frequency and cladding decay parameter respectively. Here, k_0 and are the free space wave number and propagation constant respectively.

The Chebyshev formalism based power series expression for the first higher order modal field in graded index fiber within core and cladding, can be expressed as (Patra, Gangopadhyay and Goswami, 2008)

$$\begin{aligned} \psi(R) &= a_1 R + a_3 R^3 + a_5 R^5, && R \le 1 \\ &= (a_1 R + a_3 R^3 + a_5 R^5)\frac{K_1(WR)}{K_1(R)}, && R > 1 \end{aligned} \tag{5.6}$$

Now applying Eq. (5.6) in Eq. (5.4), we get

$$\begin{aligned}
&a_1\left\{V^2(1-f(R))-W^2+V^2g\cdot\ ^2(R)\right\}+\\
&a_3\left\{8+R^2[V^2(1-f(R))-W^2+V^2g\cdot\ ^2(R)]\right\}\\
&+a_5\left\{24R^2+R^4[V^2(1-f(R))-W^2+V^2g\cdot\ ^2(R)]\right\}=0
\end{aligned} \tag{5.7}$$

Following Refs. (Patra, Gangopadhyay and Goswami, 2008; Bose, Gangopadhyay and Saha, 2011a), we choose two Chebyshev points namely $R=R_1$ and $R=R_2$ in order to find cladding decay parameter W and also a_3 and a_5 in terms of a_1. The concerned Chebyshev points are given as (Chen, 1982)

$$R_m=\cos\left(\frac{2m-1}{2M-1}\frac{\text{®}}{2}\right), \qquad m=1,\ 2,\ldots,(M-1) \tag{5.8}$$

Here, we use M =3 to get R_1=0.9511, R_2=0.5878

Using those two relevant Chebyshev points $R=R_1$ and $R=R_2$, we get two following equations

$$\begin{aligned}
&a_1\left\{V^2(1-f(R_1))-W^2+V^2g\cdot\ ^2(R_1)\right\}+\\
&a_3\left\{8+R_1^{\,2}[V^2(1-f(R_1))-W^2+V^2g\cdot\ ^2(R_1)]\right\}\\
&+a_5\left\{24R_1^{\,2}+R_1^{\,4}[V^2(1-f(R_1))-W^2+V^2g\cdot\ ^2(R_1)]\right\}=0
\end{aligned} \tag{5.9}$$

$$\begin{aligned}
&a_1\left\{V^2(1-f(R_2))-W^2+V^2g\cdot\ ^2(R_2)\right\}+\\
&a_3\left\{8+R_2^{\,2}[V^2(1-f(R_2))-W^2+V^2g\cdot\ ^2(R_2)]\right\}\\
&+a_5\left\{24R_2^{\,2}+R_2^{\,4}[V^2(1-f(R_2))-W^2+V^2g\cdot\ ^2(R_2)]\right\}=0
\end{aligned} \tag{5.10}$$

The variation of $\frac{K_1(W)}{K_0(W)}$ with 1/W in the interval $0.6\le W\le 2.5$ is fairly linear and accordingly, we apply least square fitting technique to get a linear relationship of $\frac{K_1(W)}{K_0(W)}$ with 1/W in the interval $0.6 \le W \le 2.5$ as follows (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997); Bose, Gangopadhyay and Saha, 2012c)

$$\frac{K_1(W)}{K_0(W)} = \sim \; + \frac{\quad}{W} \tag{5.11}$$

where $\sim$ = 1.034623 and   = 0.3890323

However, when W is less than 0.6, linear least square fitting technique is applied in order to make linear formulation of $\frac{K_1(W)}{K_0(W)}$ as a function of $\frac{1}{W}$ for different short intervals of W (Patra, Gangopadhyay and Sarkar, 2000). Accordingly α and β for different short intervals will be different.

Again, applying Eq. (5.6) and Eq. (5.11) in Eq. (5.5), we find

$$\begin{aligned} &a_1[2(\sim W + \quad) + W^2] + a_3[4(\sim W + \quad) + W^2] \\ &+ a_5[6(\sim W + \quad) + W^2] = 0 \end{aligned} \tag{5.12}$$

a_1, a_3 and a_5 given in Eq. (5.9), Eq. (5.10) and Eq. (5.12) will be conformable for non-trivial solution if those satisfy the following condition.

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0 \tag{5.13}$$

here,

$$A_1 = \left\{V^2(1 - f(R_1)) - W^2 + V^2 g \cdot \;^2(\mathrm{R}_1)\right\}$$

$$A_2 = \left\{V^2(1 - f(R_2)) - W^2 + V^2 g \cdot \;^2(R_2)\right\}$$

$$A_3 = 2(\sim W + \quad) + W^2$$

$$B_1 = \left\{8 + R_1^2[V^2(1 - f(R_1)) - W^2 + V^2 g \cdot \;^2(R_1)]\right\}$$

$$B_2 = \left\{ 8 + R_2{}^2[V^2(1 - f(R_2)) - W^2 + V^2 g \cdot \ {}^2(R_2)] \right\}$$

$$B_3 = 4(\sim W + \quad) + W^2$$

$$C_1 = \{24R_1^2 + R_1{}^4[V^2(1 - f(R_1)) - W^2 + V^2 g \cdot \ {}^2(R_1)]\}$$

$$C_2 = \{24R_2^2 + R_2{}^4[V^2(1 - f(R_2)) - W^2 + V^2 g \cdot \ {}^2(R_2)]\} \tag{5.14}$$

$$C_3 = 6(\sim W + \quad) + W^2$$

Initially, we can calculate the value of W for a given value of V by solving Eq. (5.13) in absence of nonlinearity by considering g=0. Moreover, using a particular value of W corresponding to a given V number, a_3 and a_5 in terms of a_1 is calculated in absence of Kerr-type nonlinearity by using any two of the three equations given by Eqs. (5.9), (5.10) and (5.12). Further, in order to evaluate the first higher order modal field in presence of Kerr-type nonlinearity for a particular value of g corresponding to a typical fiber having a specific V value, an iterative technique is adopted and iteration is continued till the convergent values of W and corresponding a_3, a_5 values in terms of a_1 are obtained.

5.3 RESULTS AND DISCUSSIONS

Here, we choose some typical step and parabolic index fibers (Bose, Gangopadhyay and Saha, 2011a) for verification of our formalism. In this context, we compare our results with available exact numerical results obtainable following Ref. (Hayata, Koshiba and Suzuki, 1987). Further, we have considered the refractive index of cladding (n_2) and basic fiber parameter $a\sqrt{\left(n_1^2 - n_2^2\right)}$ as 1.47 and 0.22 µm respectively (Mondal and Sarkar, 1996; Roy and Sarkar, 2016). The product of nonlinear refractive index n_{NL}(R) (m^2/W) and power P (W) is introduced as $n_{NL}P$, the value of which in our present study has been taken as -1.5x10^{-14} m^2 and 1.5x10^{-14} m^2 (Mondal and Sarkar, 1996; Roy and Sarkar, 2016).

In Fig. 5.1, we show the variation of cladding decay parameter with respect to Kerr-type nonlinearity for step index fibers having V numbers 2.5, 3.0 and 3.5 respectively. Similarly, in

Fig. 5.2, we represent the variation of cladding decay parameter with respect to Kerr-type nonlinearity for parabolic index fibers having V numbers 4.0, 4.5 and 5.0 respectively.

It is relevant to mention in this connection that such choice of V numbers has been made, taking into consideration that in case of step index fiber, the first higher mode cut-off value is 2.4048 and the next higher mode cut-off value is 3.8317 while those in case of parabolic index fibers are respectively 3.518 and 6.37 (Nuemann,1988). As such corresponding to the selected values of V, only fundamental and first higher order mode propagate through both step and parabolic index fibers. Incidentally, V numbers for the said kinds of fibers which correspond to W values in the interval $0.6 \leq W \leq 2.5$, $\frac{K_1(W)}{K_0(W)}$ can be linearly formulated by least square fitting technique while in case W values are less than 0.6, $\frac{K_1(W)}{K_0(W)}$ is linearly formulated for some short intervals of W (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997; Patra, Gangopadhyay and Sarkar, 2000). This has been also used in chapter 4.

In Figs. 5.3, 5.4 and 5.5, we plot variation of first higher order modal field with respect to normalised radial distance (R) in presence and in absence of Kerr-type nonlinearity for step index fibers having V numbers 2.5, 3.0 and 3.5 respectively. Similarly, in Figs. 5.6, 5.7 and 5.8, we show the variation of first higher order modal field with respect to normalised radial distance (R) in presence and in absence of Kerr-type nonlinearity for parabolic index fibers having V numbers 4.0, 4.5 and 5.0 respectively. Here, it is also seen that the effect of nonlinearity on the first higher order modal field becomes more as we move more towards cladding. Further, it is seen from the graphs that influence of Kerr-type nonlinearity becomes more effective for fibers having less V number, such influence being more prominent in case of step index fiber. The observation predicts that decrease of V number with respect to first higher order modal field leads to more nonlinear influence. In figures 5.3-5.6, we have presented our estimation by * in case of $n_{NL}P = 1.5\text{x}10^{-14}$ m^2, by + in case of $n_{NL}P = -1.5\text{x}10^{-14}$ m^2 and by ○ in case of $n_{NL}P=0$. Again, $n_{NL}P=0$ corresponds to linear fiber and here our results are seen to agree excellently with the results found in Ref. (Patra, Gangopadhyay and Goswami, 2008). Further, in case of nonlinearity used, our results also agree excellently with the results obtained by finite element based variational method (Hayata, Koshiba and Suzuki, 1987). The analysis prescribes prediction of first higher order modal field for graded fiber in presence of Kerr-type nonlinearity by means of solution of a third order determinant. The study also presents choice of appropriate V number concerned with dual-mode fiber for minimization of noise due to nonlinearity.

The literature contains the effect of nonlinearity on the first higher order modal field (Tai, Tomita, Jewell and Hasegaw, 1986; Snyder and Chen, 1990; Goncharenko, 1990; Sammut and Pask, 1990; Agrawal and Boyd, 1992). In case of few mode fibers, the modal analysis in presence of nonlinearity is implemented by using nonlinear Schrödinger equations and then adopting simulation technique (Kutluyarov, Lyubopytov, Bagmanov and Sultanov, 2017). Moreover, the fiber nonlinear Kerr coefficient of two mode fiber can be also measured by characterising the four-wave mixing components (Chen, Li, Gao, Amin and Shieh, 2012). The involved processes are lengthy and cumbersome. On the other hand, our formalism based on iterative Chebyshev technique provides prediction of the first higher order modal field in an accurate fashion. Our approach is novel in the sense that no such simple but accurate formalism for prediction of first higher order mode has been added to literature till date. The merit of our formalism leaves ample scope for its extension in study of other kinds of fiber in presence of nonlinearity.

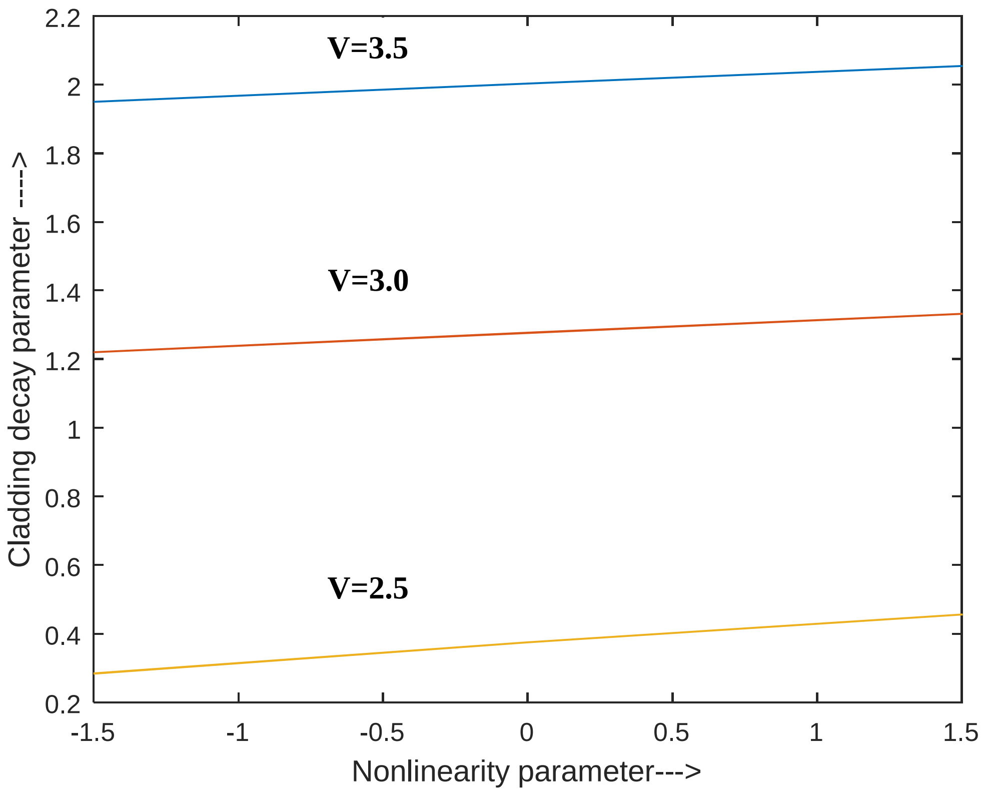

Fig. 5.1: Variation of Cladding decay parameter for first higher order mode against nonlinearity parameter $n_{NL}P$ ($\times 10^{-14}$ m^2) in case of step index fibers having different V numbers.

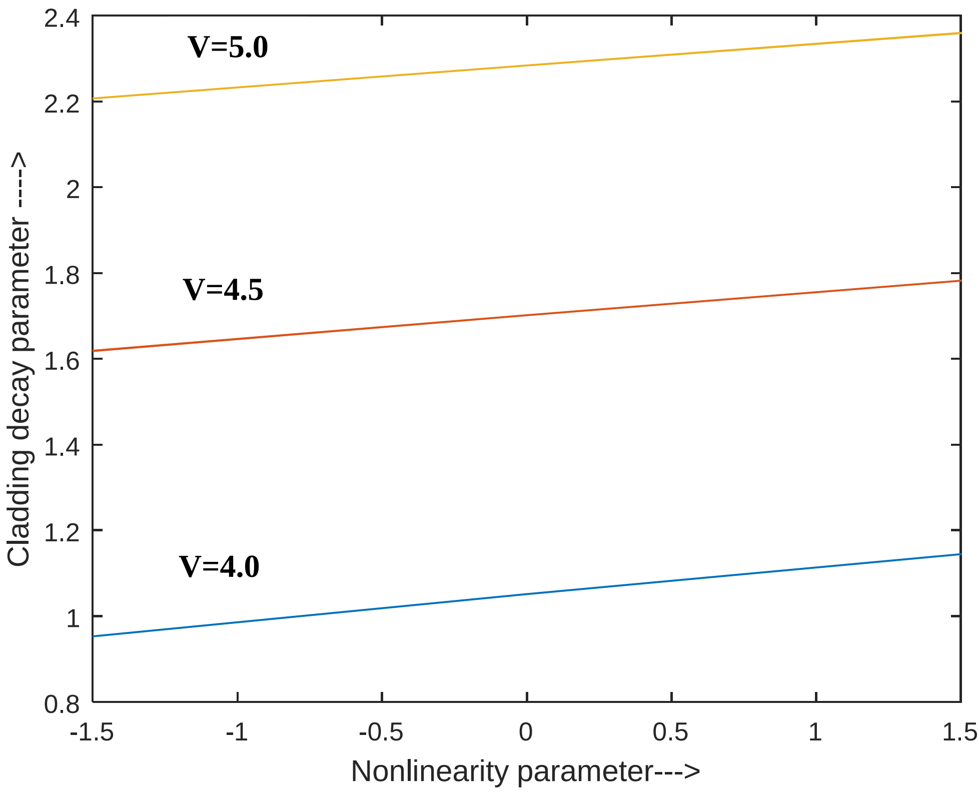

Fig. 5.2: Variation of Cladding decay parameter for first higher order mode against nonlinearity parameter $n_{NL}P$ ($\times 10^{-14}\ m^2$) in case of parabolic index fibers having different V numbers.

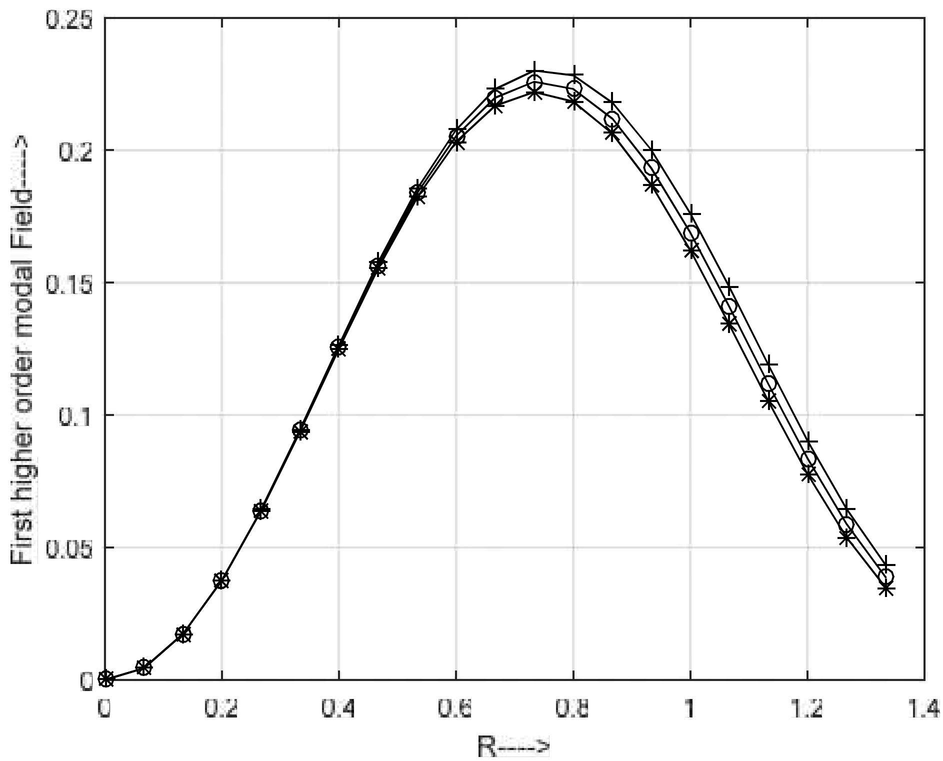

Fig. 5.3: Variation of first higher order modal field against normalised radial distance R in case of step index fiber having V=2.5 for different nonlinearity parameter $n_{NL}P$ (* for $n_{NL}P$ =1.5x10^{-14} m^2, +for $n_{NL}P$ = -1.5x10^{-14} m^2, O for $n_{NL}P$ = 0 : Our results ; ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

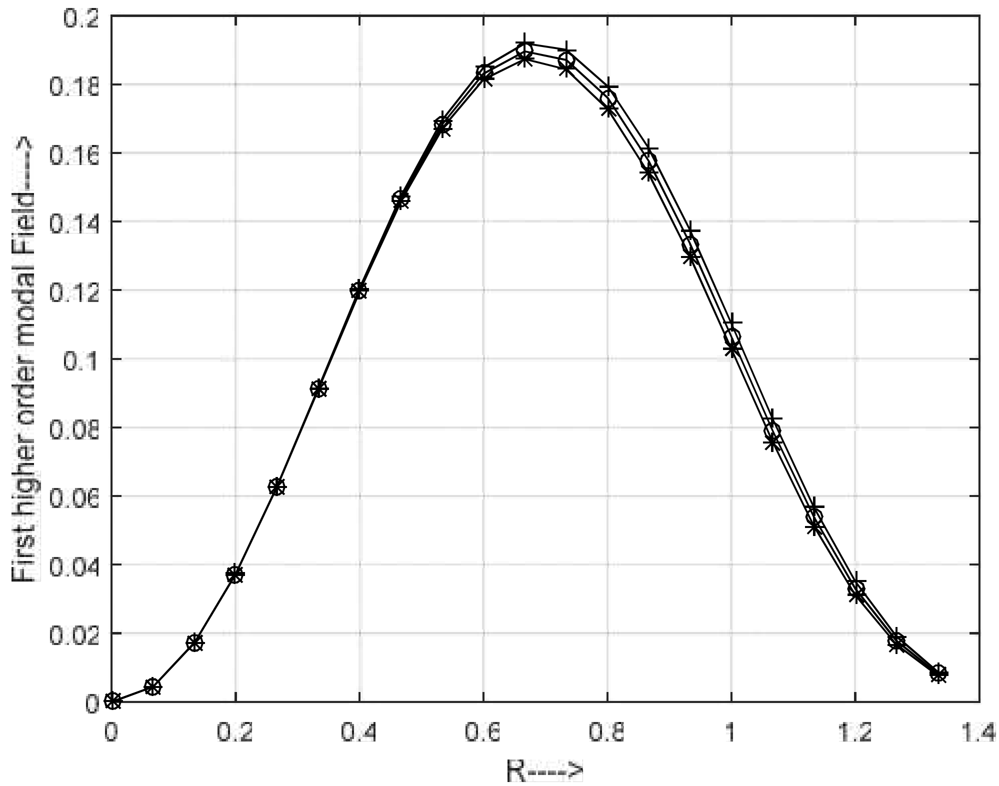

Fig. 5.4: Variation of first higher order modal field against normalised radial distance R in case of step index fiber having V=3.0 for different nonlinearity parameter $n_{NL}P$ (* for $n_{NL}P$ =1.5x10^{-14} m^2, + for $n_{NL}P$ = -1.5x10^{-14} m^2, O for $n_{NL}P$ = 0 : Our results ; ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

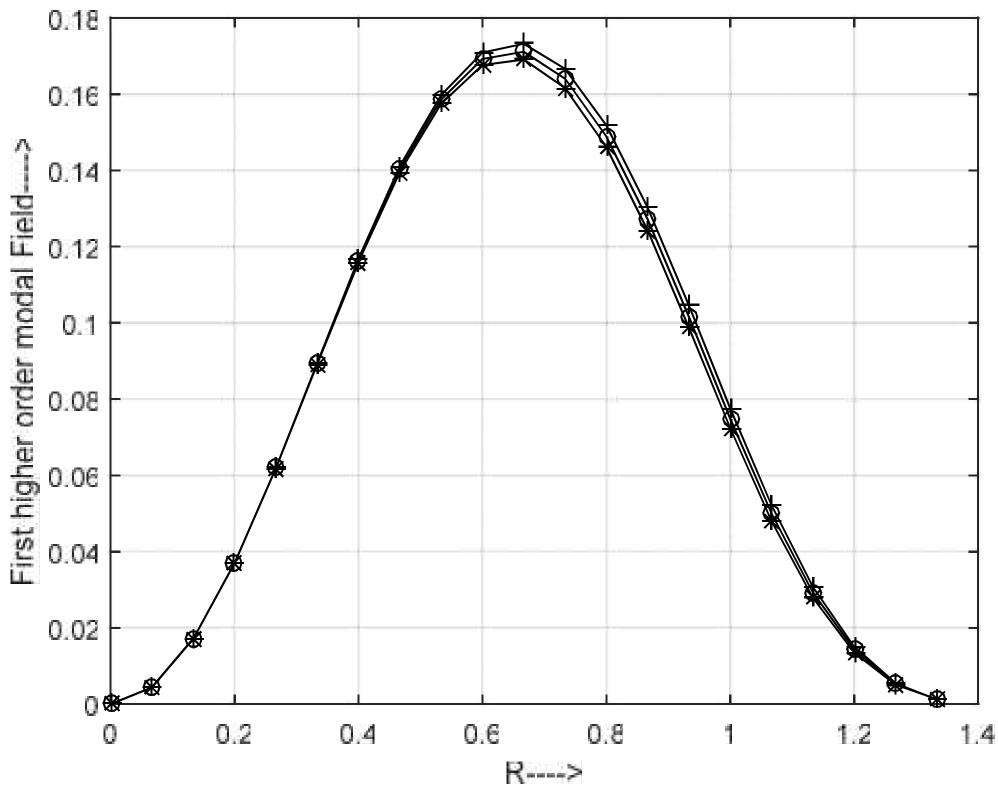

Fig. 5.5: Variation of first higher order modal field against normalised radial distance R in case of step index fiber having V=3.5 for different nonlinearity parameter $n_{NL}P$(* for $n_{NL}P$ =1.5x10^{-14} m^{2}, + for $n_{NL}P$ = -1.5x10^{-14} m^{2}, O for $n_{NL}P$ = 0 : Our results ; ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

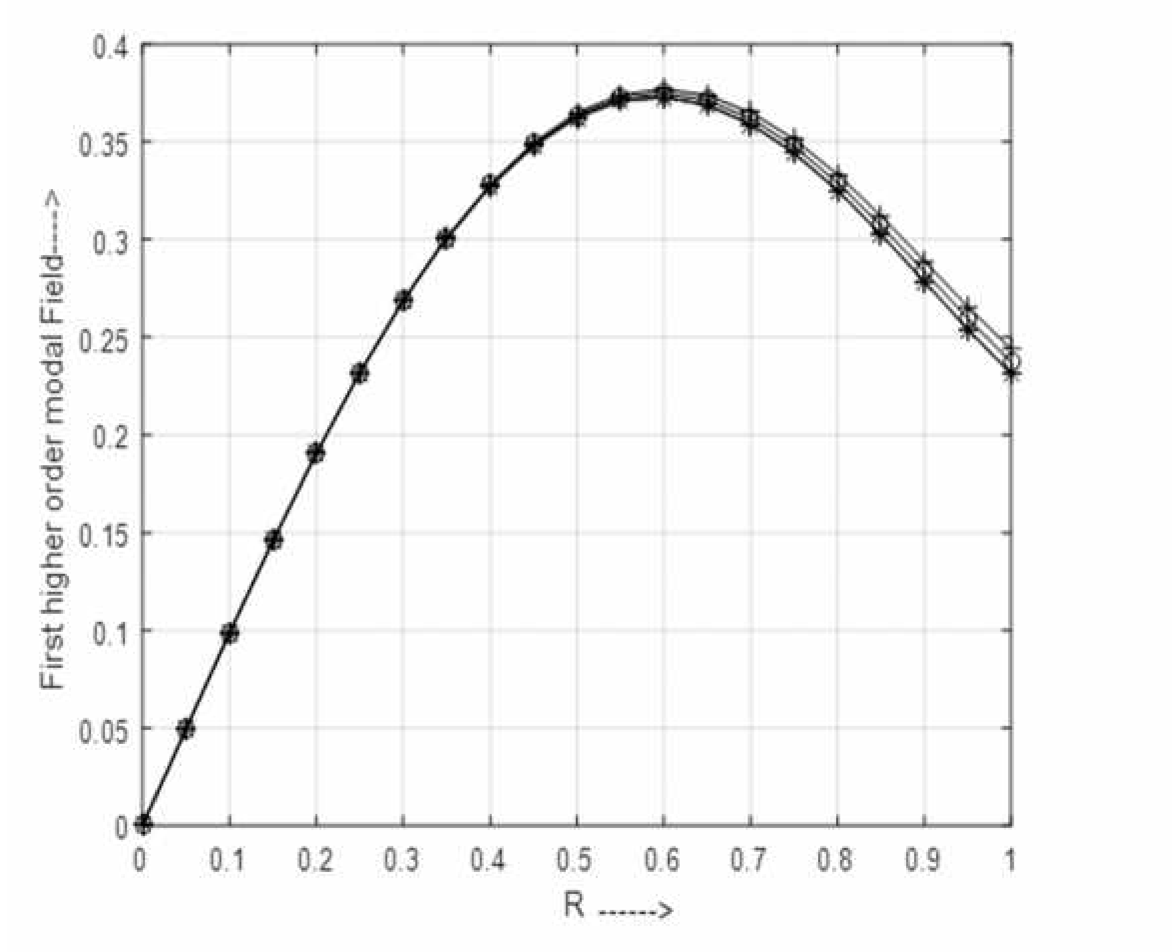

Fig. 5.6: Variation of first higher order modal field against normalised radial distance R in case of parabolic index fiber having V=4.0 for different nonlinearity parameter $n_{NL}P$ (* for $n_{NL}P = 1.5 \times 10^{-14}\ m^2$, + for $n_{NL}P = -1.5 \times 10^{-14}\ m^2$, O for $n_{NL}P = 0$ Our results; ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

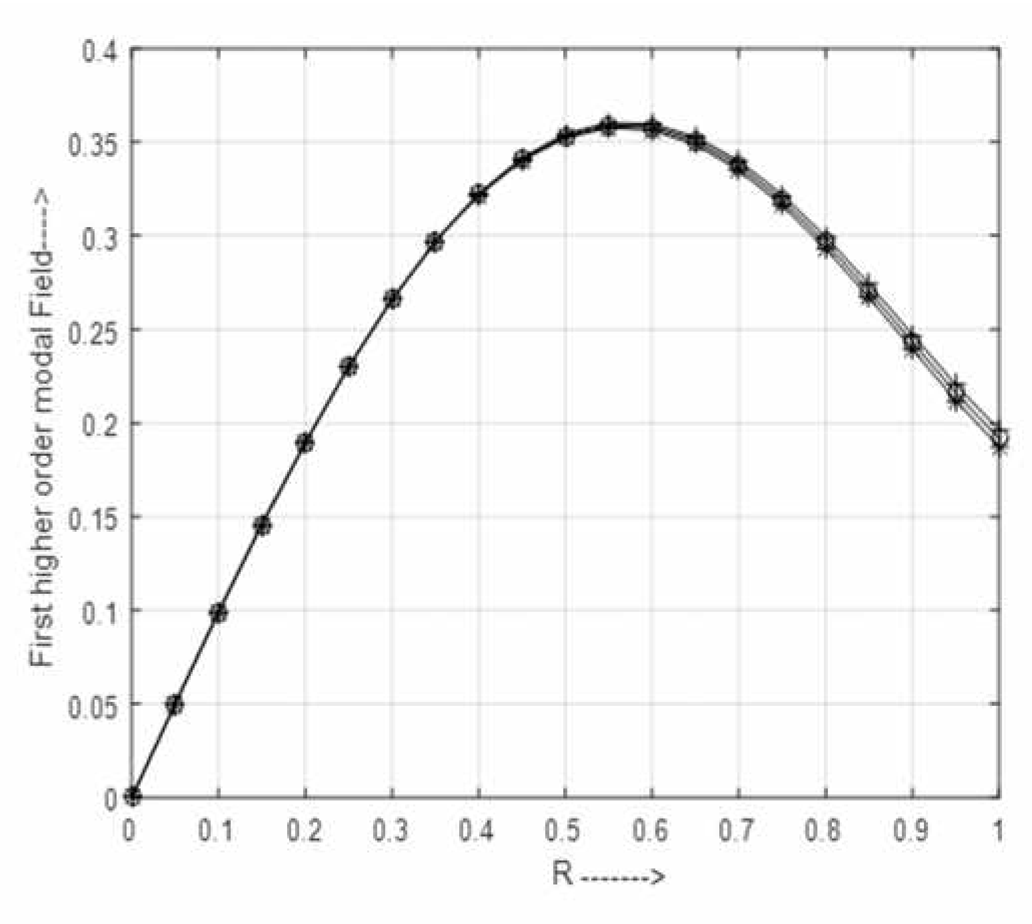

Fig. 5.7: Variation of first higher order modal field against normalised radial distance R in case of parabolic index fiber having V = 4.5 for different nonlinearity parameter $n_{NL}P$ (* for $n_{NL}P = 1.5\mathrm{x}10^{-14}$ m^2, + for $n_{NL}P = -1.5\mathrm{x}10^{-14}$ m^2, O for $n_{NL}P = 0$: Our results ; ——— : Simulated exact ones (Hayata, Kohiba and Suzuki, 1987)).

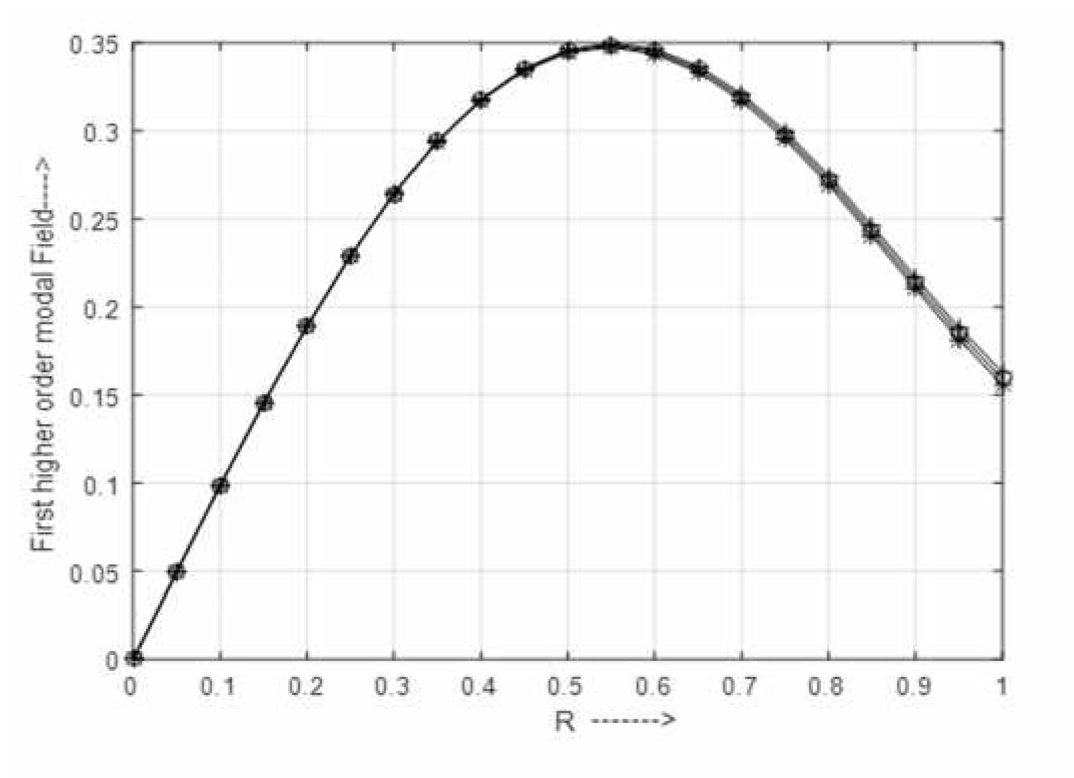

Fig. 5.8: Variation of first higher order modal field against normalised radial distance R in case of parabolic index fiber having V = 5.0 for different nonlinearity parameter $n_{NL}P$ (* for $n_{NL}P = 1.5\text{x}10^{-14}$ m^2, + for $n_{NL}P = -1.5\text{x}10^{-14}$ m^2, O for $n_{NL}P = 0$: Our results; ——— : Simulated exact ones (Hayata, Koshiba and Suzuki, 1987)).

5.4 SUMMARY

We have presented a simple but accurate formalism based on iteration for prediction of first higher order modal field for dual mode graded index optical fiber in presence Kerr-type nonlinearity. The study is carried out both in absence as well as in presence of Kerr-type nonlinearity. The analysis is based on a simple method involving Chebyshev formalism. Taking some typical step and parabolic index fibers as examples, we show that our results agree excellently with the exact results which can be obtained by applying rigorous methods. Thus, our simple formalism stands the merit of being considered as an accurate alternative to the existing cumbersome methods. The prescribed formalism provides scope for accurate estimation of different propagation parameters associated with first higher order mode in such kinds of fibers in presence of Kerr nonlinearity. The execution of formalism being user friendly, it will be beneficial to the system engineers working in the field of optical technology.

CHAPTER – 6

CONCLUSIONS

CONCLUSIONS

In chapter 1, the basics of fiber optics along with extensive literature survey concerned with review of the background and current scenario, have been presented. This chapter also comprises how the literature survey has generated the motivation for the present work. Further, this chapter also presents the objective and scope of the thesis from the standpoint of contemporary interest.

In chapter 2, the thesis contains formulation of simple analytical expressions for power transmission coefficients at the splice separately for both angular and transverse mismatches in case of single-mode dispersion shifted trapezoidal as well as dispersion flattened graded and step W fibers. Here, the series expression for fundamental mode developed by Chebyshev technique for each kind of dispersion managed fiber, is employed. Further, taking into consideration that splices are highly tolerant with respect to longitudinal separation, the investigations are restricted to the cases of transverse and angular mismatches only. This chapter also shows excellent match between our results and available exact numerical results in case of some typical trapezoidal fibers and parabolic as well as step W fibers which have been chosen as examples. The evaluations of the said parameters by this formalism involve very little computations. Such excellent predictions leave scope for system engineers to use this user friendly but accurate formalism for study of other relevant characteristics concerned with all optical technology. Thus, our formalism may prove immensely helpful in present communication system.

In chapter 3, the thesis presents analytical formulation of normalised coupling length in terms of fiber to fiber separation for a directional coupler containing two identical single-mode dispersion managed fibers. The said prescriptions have been made by using the coupled-mode theory along with Chebyshev power series expression for fundamental mode of each kind of dispersion managed fiber. The said estimations have been made for directional couplers corresponding to some typical dispersion managed fibers and the results obtained have been shown to be comparable to available numerical results in case of directional coupler formed of two identical single-mode graded index fibers. The concerned calculations require simple and little computation and as such the formalism developed will prove beneficial to technologists who are working in the field of optical technology. Moreover, the method developed generate ample scope for extension to the analysis of directional couplers, switches etc. made of other kinds of fibers. In addition present formalism with some modification can be also applied for such devices in the low V region which happen to be conducive to evanescent coupling.

In chapter 4, the thesis contains prediction of fundamental modal field of single mode graded index fiber in presence of Kerr-nonlinearity. The formalism is based on power series expression for the modal field of single mode graded index fibers by Chebyshev formalism. The method involves necessary iteration for estimation modal field of single mode graded index fiber in presence of Kerr-nonlinearity. The results found are important for the purpose of minimisation of modal noise due to such nonlinearity in the field of optical communication. Thus, the results may prove extremely important for different kinds of sensors and in the field of integrated nonlinear photonics. The accuracy of the method opens up a simple technique for estimating various propagation characteristics of Kerr-type nonlinear graded index fibers. This simple formalism leaves scope for extension of the analysis for the study of other kinds of fibers. Moreover, such type of formalism can also be tested for analysis of different kinds of fibers in presence of higher order nonlinearity including saturable nonlinearity.

Chapter 5 of the thesis deals with prediction of first higher order modal field for dual mode graded index optical fiber in presence Kerr-type nonlinearity. The study is carried out both in absence as well as in presence of Kerr-type nonlinearity. The analysis is based on a simple iterative method involving Chebyshev formalism. It has been shown that the results found in case of some typical step and parabolic index fibers as examples, agree excellently with the exact results which can be obtained by applying rigorous methods. Thus, our simple formalism stands the merit of being considered as an accurate alternative to the existing cumbersome methods. The prescribed formalism provides scope for accurate estimation of different propagation parameters associated with first higher order mode in such kinds of fibers in presence of Kerr-type nonlinearity. The execution of formalism being user friendly, it will be beneficial to the system engineers working in the field of optical technology. The accuracy of this simple formalism opens up scope for its application with necessary modification in other kinds of fiber and other types of nonlinearity as well.

The investigations made have resulted in four publications in international journal of repute. The present work can motivate one to proceed ahead with prescribed formalism in various areas involving splice losses, directional couplers, switches etc.. The formalism developed for study of influence of Kerr-type nonlinearity on single-mode as well as dual mode graded index fiber generates ample scope for extension to fibers of different kinds of refractive index profile subjected to nonlinearity of various kinds. In this respect, one needs to keep track of recent publications from laboratories where necessary infrastructural facilities are available.

APPENDICES

Appendix – A

For weakly guiding fibers, the fundamental modal field $\cdot\,(R)$ inside the core of the fiber is given by the following scalar wave equation

$$\frac{d^2\cdot}{dR^2}+\frac{1}{R}\frac{d\cdot}{dR}+\left(V^2(1-f(R))-W^2\right)\cdot\ =0, \qquad R\le 1 \tag{A1}$$

together with the boundary condition

$$\left(\frac{1}{\cdot}\frac{d\cdot}{dR}\right)_{R=1} = -\frac{WK_1(W)}{K_0(W)} \tag{A2}$$

where $V\,[=k_0 a(n_1^2-n_2^2)^{1/2}]$ and $W\,[=a(\ \ ^2-n_2^2k_0^2)^{1/2}]$ represent the normalized frequency and cladding decay parameter respectively with k_0 and being the free space wave number and propagation constant respectively. Here, $\cdot$ denotes $\cdot\,(R)$ for brevity.

The fundamental modal field in the cladding of the fiber is given as

$$\cdot\,(R)\sim K_0(WR), \qquad R>1 \tag{A3}$$

The fundamental modal field $\cdot\,(R)$ being an even function of R with $\cdot\,'(0)$ being zero and $\cdot\,(0)$ nonzero, one can approximate $\cdot\,(R)$ in the following form of Chebyshev power series (Chen,1982;Shijun,1987)

$$\Psi(R)=\sum_{j=0}^{j=M-1} a_{2j}\,R^{2j} \tag{A4}$$

For simplicity and accuracy as well, it is enough to retain terms up to j = 3 in (A4) (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997; Gangopadhyay and Sarkar, 1997a, 1998a, 1998b) whereby one obtains

$$\cdot\ (R) = a_0 + a_2 R^2 + a_4 R^4 + a_6 R^6 \qquad \text{(A5)}$$

The Chebyshev points are defined as follows (Chen,1982)

$$R_m = \cos\left(\frac{2m-1}{2M-1}\,\frac{\text{®}}{2}\right) \text{; m = 1, 2, …………………….(M-1)} \qquad \text{(A6)}$$

Clearly M=4 will give Chebyshev points appropriate for (A5) and therefore the relevant three values of R corresponding to m = 1, 2 and 3, are found as follows

R_1 = 0.9749, R_2 = 0.7818 and R_3 = 0.4338 (A7)

Employing (A5) in (A1), one obtains the following three equations corresponding to three values of R given in (A7).

$$\begin{aligned} &a_0[(V^2(1-f(R_i))-W^2)] + a_2[4 + R_i^2(V^2(1-f(R_i))-W^2)] \\ &+a_4[16R_i^2 + R_i^4(V^2(1-f(R_i))-W^2)] \\ &+a_6[36R_i^4 + R_i^6(V^2(1-f(R_i))-W^2)] = 0 \end{aligned} \qquad \text{(A8)}$$

Application of least square fitting technique over the range $0.60 \le W \le 2.5$, a long interval allowing mono-mode operation of such fibers, gives the following formulation (Gangopadhyay, Sengupta, Mondal, Das and Sarkar, 1997)

$$\frac{K_1(W)}{K_0(W)} = 1.034623 + \frac{0.3890323}{W} \qquad \text{(A9)}$$

Using (A5) and (A9) in (A2), one obtains

$$\begin{aligned}&a_0(0.3890323+1.034623W)+a_2(2.3890323+1.034623W)+\\&a_4(4.3890323+1.034623W)+a_6(6.3890323+1.034623W)=0\end{aligned}\tag{A10}$$

The condition of nontrivial solution of a_2, a_4, a_6 in terms a_0 from three equations in (A8) and one equation in (A10) is given as follows

$$\begin{vmatrix} \sim_1 & {}_1 & ¥_1 & ¢_1 \\ \sim_2 & {}_2 & ¥_2 & ¢_2 \\ \sim_3 & {}_3 & ¥_3 & ¢_3 \\ \sim_4 & {}_4 & ¥_4 & ¢_4 \end{vmatrix} = 0 \tag{A11}$$

here,

$$\sim_i = V^2(1-f(R_i))-W^2$$

$$\begin{aligned}{}_i &= 4+R_i^2(V^2(1-f(R_i))-W^2)\\ ¥_i &= 16R_i^2+R_i^4(V^2(1-f(R_i))-W^2)\\ ¢_i &= 36R_i^4+R_i^6(V^2(1-f(R_i))-W^2)\end{aligned}\tag{A12}$$

where i = 1, 2, 3 and

$$\sim_4 = 1.034623W+0.3890323$$
$${}_4 = 2+\sim_4;\quad ¥_4 = 4+\sim_4;\quad ¢_4 = 6+\sim_4$$

Employing (A11), one can evaluate W for a given value of V. Further, from the evaluated value of W for a particular V, one can obtain a_2, a_4 , a_6 in terms of a_0 by using any three of four equations given by (A8) and (A10). Therefore, the transverse field of fundamental mode for a particular value of V is found by this simple method and those are given below

$$\psi(R) = a_0(1 + A_2R^2 + A_4R^4 + A_6R^6), \qquad R \le 1$$
$$= a_0(1 + A_2 + A_4 + A_6)\frac{K_0(W_C R)}{K_0(W_C)}, \qquad R > 1 \tag{A13}$$

where, $A_{2j} = a_{2j}/a_0$; j = 1, 2, 3 and W_C is the value of W estimated by the present method.

The profile functions *f(R)* for some typical dispersion managed fibers are given as (Bose, Gangopadhyay and Saha, 2012c)

(I)
$$f(R) = 0 \qquad 0 < R \le S$$
$$f(R) = \frac{R - S}{1 - S} \qquad S < R \le 1$$
for trapezoidal fiber (Paek, 1983)

(II)
$$f(R) = \rho R^q \qquad R \le \frac{1}{C}$$
$$f(R) = \rho \qquad \frac{1}{C} < R \le 1$$
for graded W fibers (Mishra, Hosain, Goyal and Sharma, 1984)

(III)
$$f(R) = 0 \qquad R \le \frac{1}{C}$$
$$f(R) = \rho \qquad \frac{1}{C} < R \le 1$$
for step W fiber (Monerie, 1982)

Here, S represents the aspect ratio for trapezoidal fiber. Further, q denotes the profile exponent for W fiber and its value is ∞ for step type while ρ stands for the relative index depth of inner cladding having index n_i and it is given by, $\rho = (n_1^2 - n_i^2)/(n_1^2 - n_2^2)$.

The relevant values of profile functions along with cladding decay parameter for the above mentioned dispersion managed fibers corresponding to typical fiber parameters, have been presented below for ready reference.

Table – A1

Trapezoidal profile of V number 2.5 (Paek, 1983)

Chebyshev points	Value of aspect ratio S	Profile function $f(R)$	Cladding decay parameter W found by this method
R_1=0.9749 R_2=0.7818 R_3=0.4338	0.25	$f(R_1)$=0.9665 $f(R_2)$=0.7091 $f(R_3)$=0.2451	1.14944
	0.50	$f(R_1)$=0.9498 $f(R_2)$=0.5636 $f(R_3)$=0	1.54153
	0.75	$f(R_1)$=0.8996 $f(R_2)$=0.1272 $f(R_3)$=0	1.71141

Table – A2

Graded W profile of V number 3.0; the profile exponent q = 2 (Mishra, Hosain, Goyal and Sharma, 1984)

Chebyshev points	Value of C	Value of ρ	Profile function $f(R)$	Cladding decay parameter W found by this method
R_1=0.9749 R_2=0.7818 R_3=0.4338	1.5	1.4975	$f(R_1)$=1.4975 $f(R_2)$=1.4975 $f(R_3)$=0.2818	1.12677
	1.5	1.5000	$f(R_1)$=1.5 $f(R_2)$=1.5 $f(R_3)$=0.2823	1.12383

Table – A3

Step W profile of V number 2.0 (Monerie, 1982; Garth, 1989)

Chebyshev points	Value of C	Value of ρ	Profile function $f(R)$	Cladding decay parameter W found by this method
R_1=0.9749 R_2=0.7818 R_3=0.4338	2.0	1.3333	$f(R_1)$=1.3333 $f(R_2)$=1.3333 $f(R_3)$=0	0.604563
	2.0	1.2500	$f(R_1)$=1.2500 $f(R_2)$=1.2500 $f(R_3)$=0	0.655343

Appendix – B

We have the following important relations (Watson, 1944; Gradshteyn and Ryzhik, 1980; Abramowitz and Stegun, 1981)

$$\left(\circ + \text{«} + \neg\right)\int x^{\circ -1} Z_{\text{«}}(x) Z_{v}(x) dx \pm (\circ - \text{«} - \neg - 2)\int x^{\circ -1} Z_{\text{«}+1}(x) Z_{v+1}(x) dx$$
$$= x^{\circ}\left(Z_{\text{«}}(x) Z_{\neg}(x) \pm Z_{\text{«}+1}(x) Z_{\neg+1}(x)\right) \qquad \text{(B1)}$$

$$(\text{«}+2)\int Z_{\neg}^{2}(x) x^{\text{«}+2} dx = \pm(\text{«}+1)[\neg^{2} - \frac{1}{4}(\text{«}+1)^{2}]\int Z_{\neg}^{2}(x) x^{\text{«}} dx$$
$$\pm \frac{1}{2} x^{\text{«}+1}\left[\left\{x \frac{d}{dx} Z_{\neg}(x) - \frac{(\text{«}+1)}{2} Z_{\neg}(x)\right\}^{2} + \left\{\pm x^{2} - \neg^{2} + \frac{(\text{«}+1)^{2}}{4}\right\} Z_{\neg}^{2}\right] \qquad \text{(B2)}$$

$$\int x^{-\text{«}-\neg-1} Z_{\text{«}+1}(x) Z_{\neg+1}(x) dx = \mp \frac{x^{-\text{«}-\neg}}{2(\text{«}+\neg+1)}\left[Z_{\text{«}}(x) Z_{\neg}(x) \pm Z_{\text{«}+1}(x) Z_{\neg+1}(x)\right] \qquad \text{(B3)}$$

$$\int x^{\text{«}+\neg+1} Z_{\text{«}}(x) Z_{\neg}(x) dx = \frac{x^{\text{«}+\neg+2}}{2(\text{«}+\neg+1)}\left[Z_{\text{«}}(x) Z_{\neg}(x) \pm Z_{\text{«}+1}(x) Z_{\neg+1}(x)\right] \qquad \text{(B4)}$$

$$\int x Z_{0}^{2}(x) dx = \frac{x^{2}}{2}\left[Z_{0}^{2}(x) \pm Z_{1}^{2}(x)\right]$$

$$\int x Z_{1}^{2}(x) dx = \frac{x^{2}}{2}\left[Z_{1}^{2}(x) - Z_{0}(x) Z_{2}(x)\right]$$

$$\int x^3 Z_1^2(x)dx = \frac{x^4}{6}\left[Z_1^2(x) \pm Z_2{}^2(x)\right] \qquad \text{(B5)}$$

$$\int x^5 Z_1^2(x)dx = \frac{x^6}{40}\left[5Z_1^2(x) \pm 4Z_2^2(x) - Z_3^2(x)\right]$$

$$\left(\frac{1}{x}\frac{d}{dx}\right)^m \left(\frac{Z_\nu(x)}{x^\nu}\right) = (\pm 1)^m \frac{Z_{\nu+m}(x)}{x^{\nu+m}} \qquad \text{(B6)}$$

$$Z_{\nu-1}(x) - Z_{\nu+1}(x) = \pm\frac{2\nu}{x} Z_\nu(x) \qquad \text{(B7)}$$

Here, $Z_\nu(x)$ stands for either the Bessel function $J_\nu(x)$ or the modified Bessel function $K_\nu(x)$ with the upper and lower signs representing J type and K type Bessel functions respectively.

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