M.E. (Water Resources & Hydraulic Engineering) (6 Semester), 2025

(3rd Semester)

COMPUTATIONAL HYDRO DYNAMICS

(Paper- VII)

Full Marks : 100

Answer any ***four*** questions.

1. a) State different advantages and disadvantages of experimental, theoretical and computational approaches for solving non-linear problems.

   b) Explain what do you mean by "well posed" and "ill posed" problems?

   c) What do you mean by a staggered grid? Explain with examples.

   d) Define consistency with a suitable example.

   e) Give examples of hyperbolic, elliptic and parabolic partial differential equations as found in hydro dynamics.

   8+2+6+3+6= 25

2. a) Find a root of the equation $x \operatorname{Sin} x = 1$, using the bisection method in three stages.

   b) Find by Newton-Raphson method, the real root of the equation $x^4 - x = 10$. Correct to three stages.

   c) Find root of the equation $x^3 - 2x = 5$, by Regula-falsi method correct to two decimal places.

   d) Apply the Gauss elimination method to solve the equations $10x + 2y + z = 9$; $2x + 20y - 2z = -44$; $-2x + 3y + 10z = 22$.

   6+7+6+6 = 25

3. a) Deduce the C.F.L. condition. What do you mean by Courant number?

   b) Consider the function $z = e^{2x} + 2e^{y}$. Consider the point $(x, y) = (0.1, 0.1)$. Assume, $\Delta x = \Delta y = 0.01$.

   (i) Calculate the exact values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at this point.

   (ii) Calculate the same using first-order accurate forward and backward differences. Also, calculate the percentage difference compared with the exact values.

   (iii) Calculate the same using second-order accurate central differences. Also, calculate the percentage difference compared with the exact values.

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13+12 = 25

4. a) Draw the stencil diagrams for the following schemes:

   (i) First-order forward difference of $(\partial w/\partial v)_{j,k}$ with respect to x.

   (ii) Second order central difference of $(\partial v/\partial x)_{i,j}$ with respect to x.

   (iii) Second order central difference of $(\partial^2 u/\partial x^2)_{i,j}$ with respect to x.

   b) Describe FTBS, FTCS, FTFS, and Leapfrog schemes with the help of stencil diagrams.

   c) By Taylor series expansion derive the second-order accurate finite difference expressions for the term $\dfrac{\partial u}{\partial y}$ at the forward and backward central boundaries of the flow domain.

6+7+12 = 25

5. a) By Taylor series expansion show how can you obtain first-order and second-order accurate finite difference expressions for the term $\dfrac{\partial^2 u}{\partial y \partial z}$ at the forward, backward and central boundaries.

25

6. a) What do you mean by reflection boundary condition?

   b) By the Polynomial approach determine the first-order and third-order accurate forward difference expressions for the term $\partial u/\partial z$.

   c) Consider the viscous flow of water over a river bed. At a given station in the flow direction, the variation of the flow velocity (u) in the direction perpendicular to the bed (the y-direction) is given by the expression $u = 100p(2p^2 - p + 0.5)$ where $p = y/h$ and h = total depth of water = 100 cm. The dynamic coefficient of viscosity of water at 20°C (μ) = 0.001002 kg/(m·s). Values of u in m/s were measured at discrete grid points with $\Delta y = 5$ cm. Calculate the shear stress (τ_o) at the wall using the following ways using (a) first order, (b) second order and (c) third order accurate forward difference using the Polynomial approach. Finally, compare these calculated finite-difference results with the exact value of τ_o.

3+9+13 = 25