M.E. (Water Resources & Hydraulic Engineering) Examination, 2025

(2nd Semester)

COMPUTATIONAL HYDRO DYNAMICS

(Paper X)

Time : Three Hours Full Marks : 100

Answer any ***four*** questions.

1. a) Find a root of the equation $x^3 - 4x - 9 = 0$; using the bisection method in four stages.

   b) Find by Newton-Raphson's method, the real root of the equation $xe^x - 2 = 0$ correct to three decimal places.

   c) Find root of the equation $4x - \log_{10} x = 7$, which lies between 3.5 and 4 by regula-falsi method.

   d) Apply Gauss elimination method to solve the equations $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$.

   6+7+6+6 = 25

2. a) With suitable examples, explain the basic characteristics of an equilibrium problem and a marching problem.

   b) Give the physical and mathematical classification of partial differential equations with examples.

   c) Give examples of elliptic, hyperbolic, and parabolic partial differential equations as found in fluid dynamics.

   d) Describe time centred FTBS, time-centered implicit, implicit BTCS, and leapfrog schemes with the help of a Stencil Diagram.

   6+5+6+8 = 25

3. By Taylor series expansion, show how you can obtain 1st order and 2nd order accurate finite difference expressions for the terms (i) $\frac{\partial u}{\partial y}$, (ii) $\frac{\partial^2 u}{\partial y^2}$, and (iii) $\frac{\partial^2 u}{\partial x \partial y}$ at the forward, backward, and central/mixed boundaries of the flow domain.

   25

4. a) Distinguish between forward and backward difference sink techniques with different examples.

   b) Briefly discuss the errors that may be introduced due to the finite difference approximation of a partial differential equation.

   c) Discuss briefly the advantages and disadvantages of a higher-order accurate finite difference expression.

   d) Define consistency with a suitable example.

   e) Discuss briefly the advantages and disadvantages of explicit and implicit approaches.

6+6+4+4+5 = 25

5. a) Consider the function $f(x,y) = e^{(2x-1)} + e^{(3y-1)}$. Consider the point $(x,y) = (2,3)$. Assume, $\Delta x = 0.2$ and $\Delta y = 0.3$.

   (i) Calculate the exact values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at this point.

   (ii) Calculate the same using first order accurate forward and backward differences. Also, calculate the percentage difference compared with the exact values.

   (iii) Calculate the same using second order accurate central differences. Also, calculate the percentage difference compared with the exact values.

   b) Write down the Vorticity transport equation and obtain the necessary stability condition for it.

11+14 = 25

6. a) Find out the Stability condition for the 1-D heat conduction equation.

   b) Consider the viscous flow of water over a river bed. At a given station in the flow direction, the variation of the flow velocity (u) in the direction perpendicular to the bed (the z direction) is given by the expression $u = 100(3p^3 - p^2 + 0.5p)$ where $p = z/h$ and h is the total depth of water = 1 m, velocity along the flow (u) is in meter per second. The dynamic coefficient of viscosity of water at 20°C is 0.001002 kg/(m·s). Values of u were measured at discrete grid points with Δz = 4 cm. Calculate the bed shear stress using the following ways: (i) Using first order accurate forward difference, (ii) Using the second order accurate forward difference and (iii) using the third order accurate forwardd difference with the help of Polynomial approach. (iv) Finally, compare these calculated finite-difference results with the exact value of bed shear stress.

12+13 = 25