Ex./FP/I/IX/37/2018

BACHELOR OF SCIENCE EXAMINATION, 2018

(3rd Year, 1st Semester)

PHYSICS (HONOURS)

Paper - HO - 9

Time : Two hours — Full Marks : 50

(25 marks for each group)

Use a separate Answer Scripts for each group.

GROUP - A

Answer ***q.no. 4*** and any ***two*** questions from the rest.

1. (a) Find the Fourier transform of the following function

$$f(x) = a\,\delta(x) + b\delta'(x) + c\delta''(x)$$

where a, b and c are real constants and $\delta(x)$ is the Dirac delta-function (prime denotes derivative).

(b) Solve the following differential equation with the help of Laplace transformation with initial conditions $x'(0) = x(0) = 0$.

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$$

(c) Using Generating function show that

$$\cos x = J_0(x) + 2\sum_{i=1}^{\infty}(-1)^n J_{2n}(x)$$

$$\sin x = 2\sum_{i=1}^{\infty}(-1)^{n-1} J_{2n-1}(x)$$

3+3+4

(Turn over)

2. (a) Find the Laplace transform of the following function

$$f(t) = \begin{cases} \frac{1}{T} & for\, 0 < t < T \\ 1 & for \quad t > T \end{cases}$$

(b) Find the Fourier transformation of the following function : $\psi(x) = \sin\frac{n\pi x}{a}$.

(c) Prove the addition theorem of Bessel function

$$J_n(x+y) = \sum_{k=x}^{\infty} J_{n-k}(x) J_k(y)$$

Hence show that

$$J_0(x+y) = J_0(x)J_0(y) + 2\sum_{k=1}^{\infty} J_k(x)J_k(y)$$ 3+3+4

3. (a) Find the value of the following contour integral

$$\frac{1}{2\pi i}\oint \frac{c^{4z}-1}{\cosh(z)-2\sinh(z)}\,dz$$

around the unit circle C traversed in the anti-clock direction.

(b) Let u(x,y) = c^{ax} cos(by) be the real part of a function f(z) = u(x,y) + ie(x,y) of the complex variable z = x + iy where a and b are real constants and a $\neq$ 0. If the function f(z) is complex analytic everywhere in the complex plane then find a and b.

9. Consider a system of N non-interacting atoms each with magnetic moment $\vec{\mu}$, in an external magnetic field $\vec{H}$. The magnetic energy of each atom is given by,

$\in = -\vec{\mu}.\vec{H} = g\mu_0 Hm$; where m = –J, –J + 1, ...J – 1, J.

Obtain (a) the partition function and (b) the magnetization of the system. In case you cannot execute the calculation for general J, try the problem for m = –1/2, + 1/2, for partial credit. 3+2

10. Consider an air column above the surface of the earth. The air behaves like an ideal gas under uniform gravity. Its Hamiltonian is,

$$H(\vec{r}i, \vec{p}i) = \sum_{i=1}^{N}\left(\frac{|\vec{p}i|^2}{2m} + mgz_i\right),$$

where z_i is the height of the *i*th gas molecule. Calculate the density distribution of particles in the column, under the influence of gravity, at a given temperature. 5

11. Consider an ensemble of *M* identical systems at a given temperature *T*, volume *V* and chemical potential μ. Let n_i,*N* be the number of ensemble members with N particles and energy E_i. Find an expression for the probability distribution *W*{n_i,*N*}. Hence derive the expression for the grand canonical (or macro-canonical) partition function. Find explicit expressions for *T* and μ in terms of relevant physical quatities. 2+2+1

—— X ——

6. Consider a system of three non-intereacting quantum particles in a cubic box having sides of length L. Find the allowed energy states of the system between energy values $(h^2/8mL^2) \times 7$ and $(h^2/8mL^2) \times 21$ and also their degeneracies (number of microstates that correspond to each of the macrostates). 5

7. Consider an ultra-relativistic ideal gas in 3 spatial dimensions given by the Hamiltonian,

$$H = \sum_{k=1}^{N} |\vec{p}_k| c.$$

Find the entropy and the equation of state. 3+2

8. (a) Derive an expression for the volume $V_{3N}(R)$ of a 3N dimensional hypersphere of radius R.

(b) The Hamiltonian for a system of non-interacting one-dimensional harmonic oscillators is given by,

$$H\left(\{q,p\}\right)=\sum_{i=1}^{N}\left(\frac{p_i^2}{2m}+\frac{1}{2}m\omega^2 q_i^2\right)$$

Find the phase space volume between the energy values $\left(E-\frac{\Delta E}{2}\right)$ and $\left(E+\frac{\Delta E}{2}\right)$. 2+3

3. (c) Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in a Laurent series valid for 1 < |=| < 3. 4+3+3

4. Evaluate the following Brounwich integral

$$I=\int_{a-i\infty}^{a+\infty}\frac{e^{zt}}{\left(z^2+k^2\right)^2}\,dz$$

OR

$$I=\int_{a-i\infty}^{a+\infty}\frac{e^{zt}}{\sqrt{z+1}}\,dz$$

where a and b are any positive constant. 5

GROUP - B

(Statistical Mechanics - I)

Answer any ***five*** questions.

All questions carry equal marks.

5. A random walker is moving on a square grid, starting from the origin. If the probability of moving to right, left, forward and backward are equal, what is the probability of his reaching the (2,2) position after a 6-step walk. What is the most probable position after 6 steps. List all possible final positions of the walker after 6 steps. 2+2+1

(Turn over)