EX/UG/SC/PHY/TH/01/18

Bachelor of Science Examination, 2018 (New)

First Year, First Semester

Physics (Paper- CORE 1)

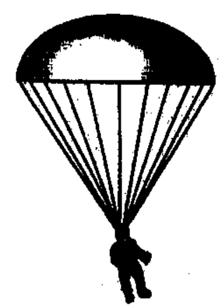
Time: Two hours | Full Marks: 50

Group: A

Answer any **five** *questions*

1. (a) A skydiver is moving vertically downward, as shown in the figure, with speed v_0 at $t = 0$ when the parachute opens. Air resistance acting on it is proportional to instantaneous speed v. Considering the skydiver including parachute as a particle of mass m and using Newton's second law set up the equation of motion and find (i) speed (ii) distance traveled for $t > 0$.

   (b) Sketch the speed-time graph showing limiting speed of the skydiver.



(2+2)+1

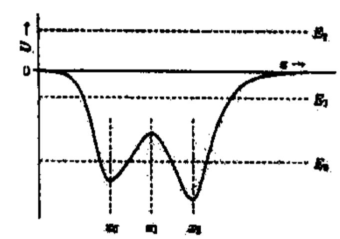
2. (a) Show that work-energy theorem is just a mathematical consequence of Newton's laws of motion. State the utilities of this theorem.

   (b) Use the above theorem to find escape velocity at the surface of the earth considering the gravitational force between two bodies is inversely proportional to the fourth power of the distance between them.

(2+1)+2

3. (a) A particle of mass m moves along the $x-$ direction under the influence of a conservative force field having potential $U(x)$. Show that total mechanical energy is a constant of this motion.

   (b) Potential energy U is given as a function of x as shown in the adjacent figure. Discuss the probable motions of a particle when the total energy of the particle is (i) E_0, (ii)E_1 and (iii) E_2.



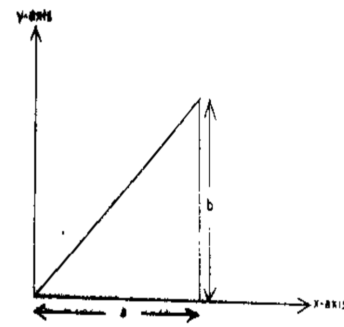
4. (a) A particle moves under the influence of a central force field. Show that (i) angular momentum o the particle is a constant of motion, and (ii) the orbit of the particle is a planer orbit.

(b) Two particles of masses m_1 and m_2 move under their mutually interacting central force. Set u the equation of motion. Explain the parameters involved in the equation.

(2+1)+2

5. (a) Define centre of mass of a rigid body.

(b) Locate the centre of mass of a triangular lamina of mass M and density ρ as shown in the figure.



1+4

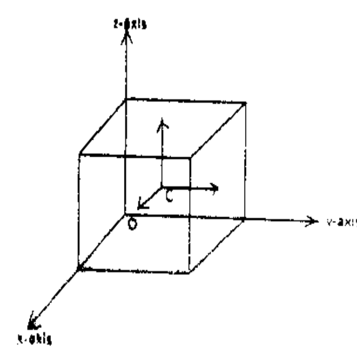
6. (a) Write down the equation of motion of a particle of mass m on a rotating frame of reference Explain the terms involved in the equation.

(b) A particle of mass m is released from rest at a height h above the Earth's surface at latitude λ Find the deviation from the vertical suffered by the particle due to Coriolis force.

(1+1)+3

7. (a) Find inertia tensor for a homogeneous cube of density ρ, mass M and side of length a. Three adjacent edges of the cube lie along the co-ordinate axes as shown in the figure.

(b) State without derivation what happens to the inertia tensor when the origin of the co-ordinate axes get shifted to the centre of mass C?



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Group B

Answer any **Two** questions

1. (a) Define a homogeneous function of degree n in x and y.

   (b) If $u(x,y)$ be a homogeneous function of degree n then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$.

   (c) If $u = \sin^{-1}\frac{x+2y+3z}{x^8+y^8+z^8}$ then using Euler's theorem show that

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -7\tan u.$$

   (d) Using Taylor's expansion expand $e^x \log(1+y)$ in powers of x and y up to terms of third degree. (1+3+4+4.5)

2. (a) Prove that $[(\vec{A}\times\vec{B})\times\vec{C}]\times\vec{D} + [(\vec{B}\times\vec{A})\times\vec{D}]\times\vec{C} + [(\vec{C}\times\vec{D})\times\vec{A}]\times\vec{B}$

   $+[(\vec{D}\times\vec{C})\times\vec{B}]\times\vec{A} = 0$

   (b) Find the constant m and n such that the surface $mx^2 - 2nyz = (m+4)x$ will be orthogonal to the surface $4x^2 + z^3 = 4$ at the point $(1,-1,2)$.

   (c) Show that div (grad r^n) $= n(n+1)r^{n-2}$ where $r = \sqrt{x^2+y^2+z^2}$. Hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$

   (d) Using divergence theorem evaluate $\int(yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k})\cdot d\vec{s}$ where S is the surface of the sphere $x^2+y^2+z^2 = a^2$ in the first octant. (2+3+3+4.5)

3. (a) Define *base vector* $(\vec{b_i})$ of any curvilinear coordinate system. Hence determine the unit vectors $(\hat{e}_i)$ of spherical polar coordinate system.

(b) Define divergence of a vector field and hence find out the form of divergence operator in the spherical polar coordinate system. Hence find out the Laplacian (∇^2) of a scalar function ϕ in the same coordinate system.

(1+3+1.5+5+2)

4. (a) Solve completely the differential equation (State the rule that you have used)

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + x^3 + \cos 2x$$

(b) A capacitor of capacitance C is allowed to discharge through a circuit of resistance R and inductance L. Write down the differential equation involving the charge q at any time t with L, C and R. Hence show that if R is sufficiently small, the charge is oscillatory and determine the period of oscillation.

(7.5+5)