MASTER OF SCIENCE EXAMINATION, 2018

(2nd Year, 1st Semester)

PHYSICS

Quantum Field Theory

Ppaer : PHY/TE/203

Time : 2 hrs | Full Marks : 40

Use a separate Answer Scripts for each group.

GROUP A

Answer any ***four*** questions. Each question carries ***five*** marks.

1. Show that the equations of motion obeyed by bosonic and fermionic operators for a quantized Schrodinger field resemble that obeyed by classical normal modes for a field of harmonic oscillators.

2. Show, by appropriately defining the number and Hamiltonian operators for a Schrodinger field, the number of particles remains conserved.

3. Let $|vac\rangle$ represent the vacuum state in a many-body quantum system. Show that the state $\hat{\Psi}^\dagger(\vec{x}_1, t)\hat{\Psi}^\dagger(\vec{x}_2, t)|vac\rangle$ is an eigenstate of the field number operator with eigenvalue 2. Interpret your results for bosons and fermions separately.

4. Starting from the position-space expression for the full (i.e., including two-particle interactions) field Hamiltonian of a field that is not subjected to any external potential, find the corresponding expression in momentum space.

5. Suppose we have a bosonic system that shows macroscopic occupation in the ground state. Assuming inter-particle contact interaction, write the full momentum-space Hamiltonian in terms of total particle number, the ground state energy and the bosonic operators for excited states only, by appropriately identifying the diagonal and off-diagonal terms.

Group – B

Answer any four questions. **4x5=20**

1. Considering the Klein-Gordon Field $\varphi(\vec{x})$ as a set of Harmonic Oscillators, write down its Fourier decomposition in terms of creation and annihilation operators. Prove that $< 0|\ \varphi(\vec{x})|\vec{p} >$ expresses the wave function of a free particle.

2. Describing the commutation relations for generalized coordinates and their conjugate momentum for a discrete system of one or more particles in quantum mechanics, describe the generalization of these relations for a continuous system which is represented by fields. Write down the correspondences of the parameters between these two systems.

3. For Lorentz transformation Λ, the quantum states of Hilbert space are implemented by unitary operator $U(\Lambda)$, which implies $U(\Lambda)|p> = |\Lambda p>$. Using this relation prove the transformation relation for an operator: $U(\Lambda)\, a_p^\dagger\, U^{-1}(\Lambda) = \sqrt{\frac{E_{\Lambda p}}{E_p}}\, a_{\Lambda p}^\dagger$.

4. Prove that $E_{\vec{p}}\delta^{(3)}(\vec{p} - \vec{q})$ is Lorentz invariant. You can use the relations of the Lorentz transformation $p_3' = \gamma(p_3 + \beta E)$ and $E' = \gamma(E + \beta p_3)$. Show that Klein-Gordon particles follow Bose-Einstein statistics.

5. Prove that $D_R(x - y) \equiv \theta(x^0 - y^0) < 0|[\varphi(x), \varphi(y)]|0 >$ is a Green function of Klein-Gordon operator. Describe the Klein-Gordon propagator using Feynman prescription.

6. Using Wick's theorem show that $< 0|T\{\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\}|0 > = D_F(x_1 - x_2)D_F(x_3 - x_4) + D_F(x_1 - x_3)D_F(x_2 - x_4) + D_F(x_1 - x_4)D_F(x_2 - x_3)$. Draw its Feynman diagram.

7. Write down the Lagrangian for a free scalar field. Using Euler-Lagrange's equation of motion for fields, find the Klein-Gordon equation. Find the dimension of Lagrangian density from the dimension of action.