M.Sc. (Physics) 2nd Yr, 1st Semester Examination, 2018.

PHY/TE/305: GENERAL RELATIVITY & ASTROPHYSICS

Max. Marks = 40. Max. Time = 2 hrs.

Answer any four Questions (4x10=40).

1. In a certain space time geometry the metric is
$ds^2 = -(1 - Ar^2)^2 dt^2 + (1 - Ar^2)^2 dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$.
(i) Calculate the proper distance along the radial line from the center $r = 0$, to a co-ordinate radius $r = R$.
(ii) Calculate the area of a sphere of co-ordinate $r = R$.
(iii) Calculate the three volume of a sphere of coordinate radius $r = R$.
(iv) Find the four volume of a four dimensional tube bounded by a sphere of co-ordinate radius R and two $t = const.$ planes separated by a time T.
$[2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}]$

2. (a) Establish a relation to find out Christoffel connections using metric tensors.
2(b). How does Einstein gravity behave in $1 + 1$ dimensions? Discuss qualitatively! [5+5]

3. What is meant by parallel transport of a vector. How do you implement the concept of parallel transport of a vector in a curved space. Show that the change dA_ν in vector A_ν under parallel displacement can be written as $dA_\nu = A^\mu \Gamma_{\mu\nu\sigma} dx^\sigma$, where as for the contravariant components the corresponding change is $dA^\nu = -\Gamma^\nu_{\mu\sigma} A^\mu dx^\sigma$. [1 + 2 + 5 + 2]

4(a). If F_{ij} is an anti-symmetric tensor, then show that the quantity S_{ijk} transforms as a covariant tensor of rank 3, where

$$S_{ijk} := \frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j}.$$

4(b). Write down the Schwarzschild solution (vaccum solution) and discuss its symmetries. Discuss why the singularity at $r = \frac{2GM}{c^2}$ is said to be only an apparent singularity. Also show that the Schwarzschild metric is *asymptotically flat.* [5 + 5]

5(a) Discuss the physical aspects of the equivalnce principle and the interpretation of the force of gravity according to Einstein.

5(b) By considering the geoesic equation or otherwise, argue (establish) that the *absence of gravitational effects* is tantamount to the existence of a coordinate system: ζ^i in which at a given point P, the metric tensor g_{ij} is the Minkowski metric and all the Christoffel symbols are zero, *i.e.*,

$$g_{ab}(\zeta_P) = \eta_{ab}, \qquad \text{and} \qquad \Gamma^a_{bc}(\zeta_P) = 0$$

(Such coordinates are known as the Riemann *normal* coordinates). [3 + 7]