Master of Science (Day) Examination, 2024

(1st Year, 1st Semester)

Physics

Paper : SC/PHY/PG/CORE/TH/104

(Statistical Mechanics)

Time : *Two hours* *Full Marks* : 40

Use separate Answer Script for each group.

GROUP—A

1. *(a)* Consider an ideal gas composed of N indistinguishable particles of mass m at a temperature T and the mean pressure P. N_g number of gas particles move freely in a volume $V = L_g^3$. N_s number of particles are absorbed on a surface with area L_s^2, forming a two-dimensional gas and $N = N_g + N_s$. The energy of an absorbed particle is $\varepsilon = \frac{p^2}{2m} - \varepsilon_0$, where p is the two-dimensional momentum and ε_0 is the binding energy per particle. Derive the classical partition functions of the free gas and of the adsorbed gas. 5

(b) Find the Gibbs free energies of the free and the absorbed gas. 5 (CO1)

(OR)

2. *(a)* State Boltmann H-theorem. Prove this theorem and show that the postulate of equal a priori probability is not valid when the system is not in equilibrium. 1+5

(b) Write down the Liouville's equation for classical system. State the properties of phase space density of a stationary ensemble in equilibrium and hence arrive the concept of canonical ensemble. 1+2+1 (CO1)

3. *(a)* In the theory of black-body radiation or lattice vibrations one assumes collection of large number of harmonic oscillators. Calculate the partition function of N such classical independent harmonic oscillator. 4

(b) Use mean field approach to derive the partition function of an non-ideal gas and hence establish the van der Waals equation of state. 4+2 (CO2)

(OR)

4. The reduce equation of state of a van der Waals gas is given by

$$\left(P_r + \frac{3}{v_r^2}\right)(3v_r - 1) = 8T_r$$

where $P_r = \dfrac{P}{P_C}; v_r = \dfrac{v}{v_C}; T_r = \dfrac{T}{T_C}$. P_C, v_C and T_C are the critical pressure, volume and temperature, respectively. Other symbols have their usual meanings. Examine the behaviour of the van der Waals system in the close neighbourhood of the critical points. Determine the critical exponents β and γ associated with order parameter and isothermal compressibility, respectively. 5+2+3 (CO2)

$$H = \beta\mathcal{H} = -\sum_{<i,j>} K\sigma_i \cdot \sigma_j - \sum_{<<k,l>>} L\sigma_k \cdot \sigma_l,$$

where $K = J / k_B T$ and $L = J' / k_B T$.

(a) Under Kadanoff decimation transformation, find connection formulae between interaction parameters, e.g., K, L and others.

(b) Argue why some parameters will remain relevant, and some will be ignorable.

(c) Simplify these transformation equations for interaction parameters K and L.

(d) Find the fixed points and identify the corresponding physical states. 3+2+2+3=10

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GROUP—B

Answer **one** from Q. Nos. 5 & 6 [CO3] and **one** from Q. No. 7 & 8 [CO4]

5. *(a)* Define *short range order* and *long range order* for spin $\frac{1}{2}$ Ising system.

(b) Implement Bragg-Williams approximation to show that the canonical partition function can be written as

$$Z(h,T) = \sum_{L=-1}^{+1} \frac{N!}{\left[\frac{1}{2}N(1+L)\right]!\left[\frac{1}{2}N(1-L)\right]!} e^{\beta N\left(\frac{1}{2}\varepsilon\gamma L^2 + hL\right)}$$

(c) Show that, in the absence of external field, (i.e., $h = 0$), there exists a temperature T_c, such that $L = \pm L_0$ for $T < T_c$ and $L = 0$ for $T > T_c$. Give explicit expression for T_c. 2+4+4=10

(OR)

6. Energy of a one-dimensional Ising chain with periodic boundary condition is given by

$$E = -J\sum_{i=1}^{N} s_i \cdot s_{i+1}$$

(a) For a chain of length 4, show explicitly the probability of different states and how the thermal average of $\mu = \sum_{i=1}^{4} s_i$ becomes zero. Thermal average is the mean value with corresponding Boltzmann weights.

(b) Introduce a small external magnetic field to show that the above mean value no longer vanishes.

(c) Show that appropriate order of relevant limits may give non-zero mean value of μ, even as the external field is withdrawn, i.e., $h \to 0$. Explain the physical significance of this process, and the meaning of spontaneous symmetry breaking. 3+2+5=10

7. The energy expression for Ising model in 1-dimension is given by,

$$E_I = -J\sum_{k=1}^{N} s_k s_{k+1} - h\sum_{k=1}^{N} s_k$$, with periodic boundary condition, $s_{N+1} = s_1$.

(a) Find relevant Transfer Matrix and hence find Helmholtz free energy.

(b) Find Magnetization per spin and check the possibility of phase transition.

(c) Argue why phase transition in this particular model is expected to depend on dimensionality of the lattice. 4+3+3=10

8. The Hamiltonian of a two-dimensional Ising model on a square lattice is defined as,

$$\mathcal{H} = -\sum_{<i,j>} J\sigma_i \cdot \sigma_j - \sum_{<<k,l>>} J'\sigma_k \cdot \sigma_l, \quad \sigma_i = \pm 1$$

where the first summation runs over nearest neighbour sites and second summation over next nearest neighbour sites. thus, the Hamiltonian multiplied by $\beta = 1/(k_B T)$ is,