M. Sc. Physics (Day) Examination, 2024

(1st Year, 1st Semester)

Physics

Paper : PHY/PG/CORE/TH/101

(Classical Mechanics)

Time : 2 *Hours* *Full Marks* : 40

Use separate Answer Scripts for different Groups.

GROUP—A

Answer questions **1 [CO2]**, **2 [CO3]**, **3 [CO5]** and **4 [CO6]** attempting either *(a)* **or** *(b)* in each.

1. *(a)* Consider a uniform triangular plate of mass M With vertices at (0, 0, 0), (a, 0, 0) and (0, a, 0). Ignore its thickness. Find the tensor of inertia about the three Cartesian axes. Discuss physical implication of the diagonal and off-diagonal elements that you find.

(OR)

(b) Consider three successive Euler rotations about x, modified y and further modified z axes by Euler angles $\pi/6$, $\pi/4$ and $\pi/6$ again. Find the resultant rotation matrix. Argue whether this final matrix can be obtained by a different set of Euler angles with same successive rotation axes. 5

2. *(a)* Consider two rectangular Cartesian axes systems, with (X', Y', Z') rotating with respect to (X, Y, Z), which is at rest. Find equations connecting position vectors and velocities in the two frames. Find expressions of pseudo forces that will be observed in the rotating frame.

(OR)

(b) Consider earth as a rotating frame. You may neglect the orbital motion around the sun. Consider a point on earth's surface at latitude and longitude 45º North and 45º East. Three equal masses, M each, are launched, one along latitude, one along longitude and one vertically up. Compare the centripetal and coriolis forces for the three masses. 5

3. *(a)* Find the fixed points and phase portrait for a particle moving in a double well potential. Comment on the stability of these fixed points.

(OR)

(b) A linear dynamical system is described by,

$$\frac{dx}{dt} = A \cdot x, \text{ where } A = \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$$

Find the phase portraits of the linear system for $a < 1$, $a > 1$ and $a = 1$; and identify the nature of fixed point at (0, 0) in each case. 5

(c) A particle of unit mass moves along x-axis under the influence of potential $V(x) = x(x-2)^2$. The particle is found in stable equilibrium at the point $x = 2$. Find the time period of oscillation of the particle.

[4+3+3=10] (CO-4)

(OR)

4. *(a)* Consider a conservative spring-mass system in configuration space with n degrees of freedom undergoing small oscillations about their respective positions of stable equilibrium. Show that the potential energy (V) and the kinetic energy (T) of the system can be expressed as :

$V = \sum_{ij} V_{ij} q_i q_j$ and $T = \sum_{ij} T_{ij} \dot{q}_i \dot{q}_j$, where the symbols have their usual meanings. Are the kinetic and potential energy matrices symmetric? Comment.

(b) Obtain the Lagrangian and the Euler–Lagrange's equations of motion for the above system. Solve the Euler–Lagrange's equations by considering a trial oscillatory solution. Hence show the eigen frequencies of such oscillations are positive definite and can be zero.

[(2+3+1)+(1+1+2)=10] (CO-4)

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4. (a) Write down Lorentz transformation equations for relative velocity of magnitude v along y direction. Hence show length contraction and time dilation.

(**OR**)

(b) Define interval ds^2 between events in STR. Explain all symbols, constants etc. When do we call an interval *(i)* spacelike, and *(ii)* timelike? Explain clearly the reason for such naming. 5

GROUP—B

1. (a) What do you mean by the term 'degrees of freedom'? How many degrees of freedom the fixed fulcrum of a simple pendulum has? What is/are the corresponding constraint equation/equations?

(b) What is 'cyclic coordinate'? If the generalized coordinate 'q_k' is cyclic, then show that its conjugate momentum is a constant of motion.

(c) A point mass is moving in a plane under the action of central force. What are the generalized coordinates that best describe the above motion? Hence obtain its Lagrangian and identify the cyclic coordinate (if *any*) in the framed Lagrangian.

(d) The dynamics of a particle is governed by the Lagrangian (L)

$L(x,\dot{x},t) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t$, where '$k$' is constant.

Obtain the equation of motion.

[(1+1+1)+(1+1)+(1+2)+2=10] (CO-1)

(**OR**)

2. (a) Obtain the equation of a parabola $y = ax^2$ (where a is a constant) in the Legendre transformed form. Reconstruct the standard equation of the parabola $y = ax^2$ from its Legendre transform form. Hence show that the Hamiltonian of a system in phase space is connected with its Lagrangian in the configuration space through Legendre transformation.

(b) The Hamiltonian of a particle of mass 'm' is represented as

$H(q,p,t) = \dfrac{p^2}{2m} + kqt$, where the symbols have usual meanings and k is constant. For the initial condition $q = 0$ and $p = 0$ at $t = 0$, if the trajectory is evolved obeying the equation $q(t) \propto t^{\beta}$, find the value of β.

(c) If F, G and S are the functions of canonical variables (q, p) and time t, show that $[F, G+S]_{q,p} = [F,G]+[F,S]$

[(2+2+2)+3+1=10] (CO-1)

3. (a) Explain the terms stable, unstable and metastable equilibriums in terms of energy diagrams $v(q)$ as a function of q. Here the symbols have usual meanings.

(b) A particle is moving under the influence of a potential given by

$V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3$, where $k, \lambda > 0$

Find the points of stable and unstable equilibriums.