*(c)* Specify and explain the symmetry of the following functions :

$$\psi(x_1, x_2) = 4(x_1 - x_2)^2$$

$$\psi(x_1, x_2, x_3) = \frac{x_1^2 + x_2^2 + x_3^3 - 1}{2x_1^3 + 2x_2^3 + 2x_3^3 + 5}$$

**[CO4]** [5+2+3]

★ ★ ★



**Ex/SC/PHY/UG/CORE/TH/11/2025**

## BACHELOR OF SCIENCE EXAMINATION, 2025

**(3rd Year, 1st Semester)**

**PHYSICS**

**PAPER : CORE/TH/11**

**( Quantum Mechanics and Applications—II )**

*Time* : *Two Hours* *Full Marks* : 40

### GROUP—A

Answer *any* **two** questions

**[CO2]**

**1.** *(a)* Explain how eigenfunctions of $L_z$ are related to Legendre polynomials. Show that $[L^2, L_z] = 0$. Here symbols have usual meanings.

*(b)* *(i)* Write down Pauli's spin matrices $\sigma_x$, $\sigma_y$ and $\sigma_z$. *(ii)* Establish the anticommutation relation between $\sigma_y$ and $\sigma_z$. *(iii)* Show that any 2 × 2 matrix can be expressed in terms of the unitary matrix and Pauli spin matrices. (2+2)+(1+2+3)

**[CO1]**

**2.** *(a)* Write down the Hamiltonian operator for a hydrogen atom. With the help of the radial part of the wave function, how do you explain the quantization of the energy in terms of the principal quantum number? Derivation is required. Given the form of the confluent hypergeometric series as follows,

$${}_1F_1(a;c;x) = \sum[(a)_\nu x^\nu]/[(c)_\nu \nu!]$$

(*b*) Calculate the expectation value of the kinetic energy, $T = p^2/2m$ with $p = -i\hbar\nabla$ for the 1*s* electron in the ground state of the hydrogen atom. (6+4)

**[CO1]**

3. Consider that a particle of mass *m* is subjected to a square well potential of depth, $-V_0$ ranging from $x = -a/2$ to $x = +a/2$. Write down time independent Schrödinger equations and form of solutions for all three regions. How do you find an equation which enables us *(i)* to count the number of the bound state for $E < 0$ and *(ii)* to explain the role of the potential for the counting of symmetric bound states? (3+7)

**GROUP—B**

1. (*a*) Consider the Schrödinger equation of a system is $H^0\psi_n^0 = E_n^0\psi_n^0$, where $\psi_n^0$ represents a complete set of orthonormal and non-degerate eigenfunctions with corresponding eigenvalues $E_n^0$. If the potential is perturbed slightly, show that the first order correction to the *n*th energy eigenvalue is $E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle$, where $H'$ is the perturbation.

   (*b*) The unperturbed ground state wave function for an infinite square well ($V(x) = \infty$) at $|x| \geq a$ and $V(x) = 0$, otherwise) is $\psi_0(x) = \sqrt{1/a}\cos(\pi x/2a)$. If the potential well is slightly perturbed by $V'(x) = \lambda x$, calculate the first order correction to the ground state energy.

   **[CO3]** (5+5)





**( OR )**

2. (*a*) Show that the lowest order relativistic correction to the Hamiltonian of Hydrogen atom is $H_r' = -\dfrac{\hat{p}^4}{8m^3c^3}$, where *p* is relativistic momentum.

   (*b*) Further, show that the correction to energy levels $E_n$ of hydrogen atom due to the relativistic effects is $E_r^1 = \dfrac{E_n^2}{2mc^2}\left[\dfrac{4n}{l+1/2} - 3\right]$.

   (*c*) Calculate the energy correction (in eV unit) due to the above effect to all energy levels of hydrogen atom corresponding to $n = 2$. **[CO3]** (5+3+2)

3. (*a*) Show that energy levels of hydrogen atom are changed by $E_z' = \mu_B g_J m_j B_{\text{ext}}$ due to weak external magnetic field, where $\mu_B$ is Bohr magneton.

   (*b*) Show the splitting of the $n = 3$ states in a weak magnetic field.

   (*c*) Discuss difference between the weak field Zeeman effect and strong field Zeeman effect. **[CO4]** [5+3+2]

**( OR )**

4. (*a*) Find the energy levels and wave functions of a system of four distinguishable spinless particles placed in an infinite potential well of size *a*. Use this result to infer the energy and the wave function of the ground state.

   (*b*) Infer the energy of the 1st excited state, if the third particle is the most massive.