Bachelor of Science Examination, 2025

(1st Year, 1st Semester)

PHYSICS

Paper : UG/MDC/TH/11/101A

(Mathematical Methods in Natural Science—I)

Time : 2 *Hours*   *Full Marks* : 40

Use separate Answer Script for both Groups.

The figures in the margin indicate full marks.

Candidates are instructed to give their answers in their own words as far as practicable.

GROUP—A

Answer *any* **one** from Questions **1** and **2**

1. [CO-1] *(i)* What do you mean by homogeneous function? If $u(x, y)$ is a homogeneous function in x and y of degree n, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$$

(ii) Determine whether the following differential is exact or inexact :

$$dF(x,y)=(2y^2-x^2)\,dx+(4xy-5)\,dy$$

If it is found to be exact, then determine $F(x, y)$.

(iii) Briefly point out the similarities, if any, between a Gaussian function and a sech function.

[(1+3)+(1+2)+2]

2. [CO-1] The Laplace equation in two-dimensional Cartesian coordinate system reads as

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \text{ where } u \equiv u(x,y).$$

(i) Mention the nature of the above equation.

(ii) Check whether $u(x,y)=\dfrac{1}{\sqrt{x^2+y^2}}$ is a possible solution of the above equation.

(iii) By using the separation of variables ansatz $u(x,y)=X(x)Y(y)$, obtain a general solution of the above equation. [2+2+5]

Answer *any* **one** from Questions **3** and **4**

3. [CO-4] Consider an experiment of tossing 3 unbiased coins at a time. The number of tails appeared is considered to be our (discrete) random variable x.

(i) Mention all possible outcomes of this experiment. Also calculate the associated probability for $x = 2$.

(ii) What is the probability that x is less than or equal to 2?

[(2+1)+1]

9. *(i)* Form an appropriate differential equation to generate the series expansion of the function $\tan^{-1} x$ about $x = 0$.

(ii) A harmonic oscillator of natural frequency ω is excited by an external force given by $F \sin \omega t$. Find the resonant term in the solution of the governing differential equation.

(4+5) **[CO 3]**

Answer *any* **one** from Questions **10** and **11**

10. Find the Fourier expansion of the function defined as $f(x)=-x$ for $-\pi \le x < 0$ and $f(x)=x$ for $0 \le x \le \pi$.

(4) **[CO 4]**

11. The two ends of a uniform wire of weight w per unit length are clamped on the wall from two points at the same horizontal level, so that the wire hangs forming a symmetric curve. The tension at the lowest point of the curve is T. Set the origin such that the coordinates of the lowest point are $(0, h)$ where $h = T/w$. Find the equation of the curve in the form $y = y(x, h)$.

(4) **[CO 4]**

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4. [CO-2] In the context of discrete distribution, the probability density function (pdf) of the Poisson distribution is given by

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \text{ where } \lambda \text{ is a parameter.}$$

(i) Show that $\langle x \rangle = \lambda$.

(ii) Prove that Poisson distribution is a limiting case of the Binomial distribution. [1+3]

GROUP—B

Answer *any* **two** questions (7×2=14)

5. *(i)* Find the unit normal to the surface $xy^3z^2 = 4$ at (–1,–1,2).

(ii) Using the divergence theorem, evaluate the surface integral

$$\iint_S (yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}), d\bar{s},$$

where S is the surface of $x^2 + y^2 + z^2 = a^2$ in the first octant. (2+5) **[CO 2]**

6. *(i)* Define base vector $(\overline{b_i})$ of any curvilinear coordinate system.

(ii) Determine the square of distance between two neighboring point $[|(d\boldsymbol{r})|^2]$ in the curvilinear coordinate system. Hence write down the displacement vector $(d\boldsymbol{r})$ in the orthogonal curvilinear coordinate system.

(iii) Draw a diagram to show the directions of the **unit vector** $\hat{e}_i$ of the cylindrical polar coordinate system **and** the corresponding **three reference planes**.

(1+4+2) **[CO 2]**

7. *(i)* Define the orthogonal matrix. Show that the product of two orthogonal matrix is also orthogonal.

(ii) Consider the matrix $A = \begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$. Find the eigenvalues and eigenvectors and hence construct a matrix U which can diagonalize the matrix A. Verify that U is a unitary matrix and compute UAU^{-1}. (2+5) **[CO 2]**

GROUP—C

Answer *any* **one** from Questions **8** and **9**

8. *(i)* Show that the Wronskian formed from two independent solutions of a linear ordinary differential equation of the second order is non-zero.

(ii) Find the second order differential equation whose solutions are x and e^x. (4+5) **[CO 3]**