(b) The Big Bang model does not occur in only λ dominated Universe. Explain it.

(c) Calculate the age of the Universe for matter and radiation dominated era separately. Why a matter-dominated universe is older than a radiation-dominated universe? [4+2+4]

7. CO4 *(a)* Briefly discuss the model for Expanding Universe.

(b) Is it possible for an open Universe to evolve into a closed one or vice verse? What is dark energy?

(c) Using the Einstein de-Sitter model $(\Omega_{m_0} = 1)$, find out the solution for scale factor (a). Further, calculate the age of the Universe for $h = 0.7$.

(d) In reversible adiabatic expanding Universe scenario, find the relation between rate of energy density and pressure. Further, explain the obtained relation. [2+2+3+3]

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MASTER OF SCIENCE EXAMINATION, 2025

[2nd Year, 1st Semester (Day)]

PHYSICS

PAPER : PHY/PG/CBS/TH/204

(General Relativity and Cosmology—I)

Time : *Two Hours* *Full Marks* : 40

Use separate Answer Script for both Group—A and Group—B.

GROUP—A

Answer *any* **one** from Questions **1** and **2** and *any* **one** from Questions **3** and **4**.

1. CO1 *(a)* Show that the following equation

$$\frac{d^2x^\lambda}{dS^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{dS}\frac{dx^\nu}{dS} = 0$$

is form invariant (covariant) under a general coordinate transformation.

(b) Show that the motion of a freely falling body in an arbitrary Gravitational field endowed with the metric tensor $g_{\mu\nu}$ is a curve of extremum length (or a geodesic).

(c) Given

$$\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\mu_{\nu\lambda} A^\lambda$$

then show that

$$\nabla_a A_b = \partial_a A_b - \Gamma^c_{ab} A_c \qquad [3+4+3]$$

2. CO1 *(a)* Transform the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$ under a general coordinate transformation.

(b) Show that the ordinary partial derivative of a scalar transforms as a tensor, however, the ordinary derivative of a vector V^μ does not transform like a tensor. Find out generalize the notion of derivatives to the vectors (and hence to the tensors of any general rank) so that it transforms as a tensor.

(c) If the covariant derivative is metric compatible and torsion free, then an auto parallel curve is the curve of extremum length.

3. CO2 *(a)* Find how many independent components does the (Riemann) curvature tensor have in dimensions 3 and 4?

(b) Let $g \equiv \det\{g_{\mu\nu}\}$ denote the metric tensor. Then using the standard definition of covariant derivative, in terms of the Christoffel connection $\Gamma^\alpha_{\beta\sigma}$, establish the following results :

(i) $g_{,\mu} = g^{\alpha\beta} g g_{\alpha\beta,\mu}$

(ii) $\Gamma^\alpha_{\alpha\beta} = (\ln(\sqrt{-g}))_{,\beta}$ [4+6]

4. CO2 *(a)* A static space-time has line element given by :

$ds^2 = -e^{2\phi} dt^2 + h_{ij} dx^i dx_j$, where ϕ and h_{ij} are functions of x.

Show that $\Gamma^0_{0i} = \partial_i \phi$ and also evaluate the components $\Gamma^0_{00}, \Gamma^i_{00}$ and Γ^0_{ij}.

(b) In an n dimensional space-time, how many independent components does the metric tensor $g_{\alpha\beta}$ have? [7+3]

GROUP—B

Answer *any* **two** questions (10×2=20)

5. CO4 *(a)* Derive the Friedmann equation

$$H^2(a) = H_0^2(\Omega_{ro} a^{-4} + \Omega_{mo} a^{-3} + \Omega_{ko} a^{-2} + \Omega_{\lambda o})$$

(b) Present the general expression for the age of the Universe using the above Friedmann equation.

(c) Calculate the present day mass density of matter where $\Omega_{m_0} = 0.3$. The value of $G = 6.67 \times 10^{-11}$, in SI unit and Hubble parameter $h = 0.7$ is given. [6+2+2]

6. CO3 *(a)* State the Cosmological principle and Hubble's law and show that the cosmological principle requires an expanding Universe to obey Hubble's law.