

Ex/SC/INSC/PG/CORE/TH/MI 101/2024

M. Sc. INSTRUMENTATION EXAMINATION, 2024

(1st Year, 1st Semester)

ADVANCE MATHEMATICS AND COMPUTER PROGRAMMING

Time : 4 Hours

Full Marks : 80

Answer from both the Parts.

(Use separate answer scripts for each Part)

PART—I

Answer *any four* questions :

1. (a) Expand the following function in Taylor series about the point $z = 3$; and find the radius of convergence of the series

$$f(z) = \frac{1}{z(z-2)}$$

(b) Prove that $\int_0^{\infty} \frac{\sin mx}{x} dx = \frac{\pi}{2}$. 4+6=10

2. (a) Using residue theorem, evaluate $\oint \frac{e^z}{z(z-1)^2}$ around the contour defined as $|z| = 2$.

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(2)

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases} \quad 4+6=10$$

(7)

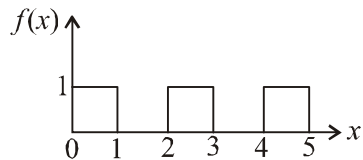
(b) Write a general program in C-Language to find the root of $x^3 - 8x - 4 = 0$ between $x = 3$ to $x = 4$ through Newton-Raphson's method. 5+5=10

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3. (a) The graph of the function $f(x)$ describes as

$$f(x) = \begin{cases} 1 & \text{for } 2n \leq x \leq 2n+1 \\ 0 & \text{for } 2n+1 \leq x \leq 2n+2 \end{cases}$$

is shown in figure. Find the Laplace transform of the function $f(x)$.



(b) Find the inverse Laplace transform of the following

$$\text{function } g(s) = \frac{1}{s^2(s+1)}. \quad 6+4=10$$

4. (a) Deduce Cauchy Riemann condition for analytic function.

(b) Let $u(x, y) = x + \frac{1}{2}(x^2 + y^2)$ be the real part of an analytic function $f(z)$ of the complex variable $z = x + iy$. Find the imaginary part of $f(z)$.

(6)
Group—B

5. Write a general program in C-Language to solve the initial value problem $\frac{dy}{dx} = 3x^2 + 1$ with $y(1) = 2$ at $x = 5$ with Euler method. Comment on the source of error in this process.
7+3=10
6. A data file named as 'data.txt' stored in your working directory containing ten pair of data (x vs. y) which is expected to be fitted with an equation $y = mx + c$. Write a general program in C-Language which will read all the data from the given file and calculate the values of m and c . The values will be printed on the video console.
7. Eight classes are defined with their upper and lower class boundaries along with the frequency of each class. This tabular data is stored in a file : 'inform.dat' at the current working directory. Write a general program in C-Language to collect the data from the above mentioned file and calculate average deviation, standard deviation and quartile deviation. Print the results on the video console.
8. (a) Write a general program in C-Language to find the integration $\int_0^9 x^3 - 4x \, dx$ with Simpson's 1/3rd rule.

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(3)

- (c) Find the Laurent's series expansion for the following function $\frac{1}{(z+1)(z+3)}$ in the region $1 < |z| < 3$.

2+4+4=10

5. (a) Find the Fourier transform of the following function

$$g(x) = \begin{cases} 1 - \left| \frac{x}{2} \right| & , \quad -2 \leq x \leq 2 \\ 0 & , \quad elsewhere \end{cases}$$

- (b) Consider the differential equation $\frac{dy}{dt} + ay = e^{-bt}$ with the initial condition $y(0) = 0$. Find the Laplace transform $Y(s)$ of the solution $y(t)$.

- (c) State the Dirichlet's conditions of Fourier series.

4+4+2=10

6. (a) Evaluate the value of the integral $\int_{-\infty}^{+\infty} \frac{dx}{(1+x^4)}$.

- (b) Consider the differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$

if $x = 0$ at $t = 0$ and $x = 1$ at $t = 1$. Find the value of x at $t = 2$.

6+4=10

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(4)

7. (a) A periodic function $f(x)$ of period 2π is defined in the interval $-\pi < x < +\pi$ as $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ +1 & 0 < x < \pi \end{cases}$; expand the function in Fourier series.
- (b) Find the value of $\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos x \delta(\sin x) dx$ using the property of delta function.
- (c) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ using contour integration.
- 3+3+4=10

PART—II

Answer *any four* questions taking *two* from each Group.

10×4=40

Group—A

1. (a) Obtain the condition for the convergence of Newton-Raphson method for finding root of an algebraic or transcendental equation $f(x) = 0$.
- (b) Show that in Newton-Raphson iteration method for finding root of an equation, the error at each iteration step is proportional to the square of the error at previous step.
- (c) Find the number of significant figures in $V_A = 103.3267$ when its absolute error is $E_a = 0.25 \times 10^{-3}$, where V_A is the approximate value of the number.
- 4+3+3

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(5)

2. (a)

x	0	1	2	3
$f(x)$	1	2	1	10
- (i) Considering the above data, construct the forward finite difference table.
- (ii) Hence find the polynomial $f(x)$ which satisfies the above data and find the value of $f(2.5)$.
- (iii) Also find the value of $f(2.5)$ using Newton's forward difference interpolation formula.
- (b) Find the relation between shift operator E and forward difference operator Δ .
- 8+2

3. Derive the general Gauss-Legendre Quadrature Formula for

equidistance ordinates for the integral $\int_a^b f(x) dx$. Hence deduce the Composite Simpson's One-third Rule for numerical integration.

6+4

4. (a) Solve the following system of simultaneous linear equations using Gauss-elimination method correct up to two decimal places.

$$5x + 3y + 7z = 4$$

$$x + 5y + 2z = 2$$

$$7x + 2y + 10z = 5$$

- (b) Compute one root of $e^x - 3x = 0$, correct to three significant digits which lies between 1 and 2 using bisection method.
- 6+4

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