BACHELOR OF SCIENCE EXAMINATION, 2018

(2nd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 4.3

[Analysis - II]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols and Notations have their usual meanings)

Answer any *five* questions.

1. (a) If $f:[a,b]\to[a,b]$ be a continuous function then prove that there exists at least one point c such that $f(c)=c$, Is it possible to have more than one such point ? Answer with justification.

(b) Determine the values of a, b, c, so that

$$\frac{ae^{x}-b\cos x+ce^{-x}}{x\sin x}\to 2 \text{ as } x\to 0.$$ 5+5

[*Turn over*]

2. (a) Suppose that a function *f* is derivable on a closed interval $[a,b]$ and $f'(a), f'(b)$ are of opposite sign. Then prove that there exists at least one point c in (a,b) such that $f'(c)=0$

(b) Let the function *f* be defined by

$$f(x)=\begin{cases} x^m \sin\dfrac{1}{x} & , \text{ when } x\neq 0 \\ 0 & , \text{ when } x=0 \end{cases}$$

Then show that *f* is derivable at $x=0$. Also determine m, when $f'(x)$ is continuous at $x=0$. 5+5

3. (a) Prove that a function $f:[a,b]\to\mathbb{R}$ is of bounded variation on $[a,b]$ if and only if $f(x)$ can be expressed as the difference of two increasing functions. Is a function $f:[a,b]\to\mathbb{R}$ of bounded variation, *R*-integrable on $[a,b]$? Justify your answer.

(b) The function $f:[0,1]\to\mathbb{R}$ is defined by

$$f(x)=x\cos {\pi}/{2x}, \quad 0<x\leq 1$$
$$= \quad 0 \quad , \quad x=0$$

Test whether *f* is of bounded variation or not on $[0,1]$. 6+4

[*Turn over*]

4. (a) If $f:[a,b]\to\mathbb{R}$ is a bounded function defined on $[a,b]$ and if f is continuous on $[a,b]$ except for a finite number of points in $[a,b]$ then show that f is Riemann Integrable on $[a,b]$.

(b) Let $f:[0,1]\to\mathbb{R}$ be defined by $f(0)=0$

$f(x)=\frac{1}{a^n}, \ \frac{1}{a^{n+1}}<x\le\frac{1}{a^n}$ for $n=0,1,2.....$ where $a>1$. Then show that f is $\mathbb{R}$- integrable on $[0,1]$

5. (a) Test the convergence of

(i) $\int_0^1 x^{n-1}\log x\,dx$

(ii) $\int_0^\infty\left(\frac{1}{1+x}-e^{-x}\right)\frac{dx}{x}$

(b) Examine the convergence of $\int_0^{\pi/2} x^m \text{cosec}^n x dx$. 6+4

[*Turn over*]

6. (a) Show that $\left|\int_p^q \frac{\sin x}{x}\,dx\right| \le \frac{2}{p}$; if $q > p > 0$.

(b) Prove that $\int_0^\infty \frac{\sin x}{x}\,dx$ is convergent but not absolutely convergent. 3+7

7. (a) When a function f is said to satisfy Lipschitz Condition in $[a,b]$? Prove that if a function f satisfies Lipschitz condition in $[a,b]$, then f is of bounded variation on $[a,b]$.

(b) Boundedness of f' is not necessary for f to be of bounded variation — Justify with an example. Also give an example of a discontinuous function which is of bounded variation. 6+4

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