BACHELOR OF SCIENCE EXAMINATION, 2018

(2nd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 4.2

[Differential Equations - III]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols and Notations have their usual meanings)

Use a separate Answer-Script for each part.

Part - I

(Marks - 25)

Answer any *two* questions from Question No. 1 to 3 and any *one* from Question No. 4 and 5.

1. (a) Show that the general solution of the PDE $Pp+Qq=R$, where P, Q, R are given continuously differentiable functions of x, y and z (not vanishing simultaneously) is $f(u,v)=0$, where f is arbitrary and $u(x,y,z)=c_1, v(x,y,z)=c_2$ are the solutions of $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$. 4

[*Turn over*]

(b) Form a PDE by eliminating the arbitrary constants a and b from $(x-a)^2+(y-b)^2+z^2=c^2$. Find the singular integral of the PDE. Also find the general integral of the PDE by assuming $b=a$. 4

(c) Solve the PDE :

$\frac{\partial^2 z}{\partial y^2}=z$, given that when $y=0, z=e^x$ and $\frac{\partial z}{\partial y}=e^{-x}$.

2

2\. (a) Describe the charpit's method of solving a first order non-linear PDE of the form $f(x, y; z; p, q)=0$. 7

(b) Solve the PDE :

$(x^2-y^2-z^2)\,p+2xy\,q=2xz$. 3

3\. (a) Find the complete integral of the PDE :

$p^3+q^3-3pqz=0$. 4

(b) Form a PDE by eliminating the arbitrary function f from

$f(x+y+z, x^2+y^2+z^2)=0$ 3

(c) Solve the PDE $p_1^2+p_2^2+p_3=1$ by Jacobi's method.

3

[*Turn over*]

4. Find the solution of a PDE of the form $z = px + qy + f(p, q)$. Hence find the complete integral of the PDE. Also find the singular integral of the PDE. 5

5. (a) Solve : $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 3x^2 y$ 3

(b) Classify the following PDEs : 2

(i) $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$

(ii) $\frac{x^2 \partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$

Part - II

(Marks - 25)

Answer Question No. 6 and any *three* from the rest.

6. Define extremal of a functional. 1

7. Derive PDE for determining the extremals of the functional

$$I\left[z(x, y)\right] = \iint_D F\left(x, y, z, z_x, z_y\right) dxdy.$$ 8

[*Turn over*]

8. Define proper field. Using the method of variational calculus, solve the problem of Brachistochrone. 1+7

9. Examine the existence of strong extremum of the functional

$$I\left[y(x)\right]=\int_0^a y'^3 dx,\ y(0)=0,\ \ y(a)=b,\ a,b>0. \qquad 8$$

10. (a) State Legendre's condition for determining extremum of the functional

$$I\left[y(x)\right]=\int_{x_0}^{x_1} F(x,y,y')\,dx,\ y(x_0)=y_0,\ \ y(x_1)=y_1.$$

(b) Find the extremal of the functional

$$I=\int_{x_0}^{x_1}\left(y'z'+2y^2+2z^2\right)dx \quad \text{with} \quad y(0)=0,\ z(0)=0$$

and the point (x_1, y_1, z_1) moves over the fixed plane $x = x_1$.

(c) State fundamental lemma of the calculus of variation.

3+4+1

11. Find the curves on which the following functional attain extremum $I=\int_{-2}^{2}\left(1+y'^2\right)^{\frac{1}{2}}dx$ subject to the conditions $y\le x^2,\ y(-2)=y(2)=3$.

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