BACHELOR OF SCIENCE EXAMINATION, 2018

(2nd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 4.1

[Vector Analysis]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols and Notations have their usual meanings)

Answer any *five* questions.

1. (a) Find the equations for the tangent plane and normal line to the surface $z = x^2 + y^2$ at the point $(2, -1, 5)$.

(b) Show that $\overline{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = C,$ where C is a constant.

(c) Prove that

$$\overline{\nabla}\left(\overline{A}\cdot\overline{B}\right) = \left(\overline{B}\cdot\overline{\nabla}\right)\overline{A} + \left(\overline{A}\cdot\overline{\nabla}\right)\overline{B} + \overline{B}\times\left(\overline{\nabla}\times\overline{A}\right) + \overline{A}\times\left(\overline{\nabla}\times\overline{B}\right)$$

3+3+4=10

[*Turn over*]

2. (a) Let u and v be differentiable functions of x and y. Show that a necessary and sufficient condition that u and v are functionally related by the equation $F(u,v)=0$ is that $\overline{\nabla}u \times \overline{\nabla}v = \overline{0}$.

(b) Prove that $\left(\overline{v} \cdot \overline{\nabla}\right)\overline{v} = \frac{1}{2}\overline{\nabla}v^2 - \overline{v} \times \left(\overline{\nabla} \times \overline{v}\right)$

(c) Evaluate $\overline{\nabla}^2\left(\ln \overline{r}\right)$ 5+3+2

3. (a) If $\overline{A} = 2yz\hat{i} - (x+3y-2)\,\hat{j} + \left(x^2+z\right)\hat{k}$. Evaluate $\int_s \overline{\nabla} \times \overline{A} \cdot \overline{ds}$ over the surface of intersection of the cylinders $x^2+y^2=a^2$, $x^2+z^2=a^2$, which is included in the first octant.

(b) Prove that,

$$\int_v \frac{dv}{r^2} = \int_s \frac{\overline{r}.\overline{n}}{r^2}\,ds$$ 7+3

4. (a) State and prove Divergence theorem.

(b) If ϕ and ψ are harmonic in v and $\frac{\partial\phi}{\partial n} = \frac{\partial\psi}{\partial n}$ on s, show that $\phi = \psi + C$, C is a constant. 7+3

[*Turn over*]

5. (a) Evaluate $\int_s \left(\overline{\nabla} \times \overline{A}\right) \cdot \hat{n}\, ds$, where $\overline{A} = \left(x^2 + y - 4\right)\hat{i} + 3xy\,\hat{j} + \left(2xz + z^2\right)\hat{k}$ and s is the surface of the paraboloid $z = 4 - \left(x^2 + y^2\right)$ above xy-plane.

(b) Let s be a closed surface and let $\overline{r}$ denotes position vector of any point (x, y, z) measured from an origin O. Find $\int_s \frac{\overline{n} \cdot \overline{r}}{r^3}\, ds$ when O lies inside s. 6+4

6. (a) Prove that,

(i) $\frac{d\hat{t}}{ds} = k\hat{n}$

(ii) $\frac{d\hat{b}}{ds} = -\tau\hat{n}$

(iii) $\frac{d\hat{n}}{ds} = \tau\hat{b} - k\hat{t}$

(b) Show that a necessary and sufficient condition for the tangent to the curve makes a constant angle with the fixed line is $\sigma/p =$ constant. 5+5

[*Turn over*]

7. (a) Find the divergence of a vector point function $\bar{f}(u,v,w)$ in terms of orthogonal curvilinear co-ordinate.

(b) Express the vector $\bar{A} = z\,\hat{i} - 2x\,\hat{j} + y\,\hat{k}$ in cylindrical co-ordinates. 5+5