Ex/Math/12/4S/55/2018 (Old)

BACHELOR OF SCIENCE EXAMINATION, 2018

(1st Year, 2nd Semester)

MATHEMATICS (Subsidiary)

Paper - 4S

(Calculus - II)

[Old Syllabus]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Part - I

(30 marks)

Answer any *three* questions. 10×3=30

1. (a) Using $\in$-δ definition show that

$$\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2}=0$$

[*Turn over*]

(b) If $x^2 = vw,\ y^2 = uw,\ z^2 = uv$ and

$f(x, y, z,) = \phi(u, v, w)$ then show that

$$xf_x + yf_y + zf_z = u\phi_u + v\phi_v + w\phi_w \qquad 5+5$$

2. (a) Examine the differentiability for the function

$$f(x, y) = \begin{cases} \dfrac{x^3 + y^3}{x - y} & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

at (0, 0).

(b) If p, q, r are the roots of the following equation in λ, $\dfrac{x}{a+\lambda} + \dfrac{y}{b+\lambda} + \dfrac{z}{c+\lambda} = 1$

then show that $\dfrac{\partial(x, y, z)}{\partial(p, q, r)} = -\dfrac{(p-q)(q-r)(r-p)}{(b-c)(c-a)(a-b)}$

5+5

3. (a) Find the moment of inertia of a uniform rectangular lamina about a line through its centre and parallel to one of its edges.

(b) Evaluate $\iiint xyz\ dxdydz$, the volume of integration being the positive octant of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} \leq 1$. 5+5

[*Turn over*]

4. (a) If $u = a^3x^2 + b^3y^2 + c^3z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the maximum or minimum value of *u*.

(b) Using a double integral, show that

$$\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)} \quad m, n > 0$$ 5+5

Part - II

(20 marks)

Answer any *two* questions : 10×2=20

5. Solve :

(a) $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xy\, e^{xy^2} - 3y^2\right)dy = 0$

(b) $xy\,dx + \left(2x^2 + 3y^2 - 20\right)dy = 0$ 5+5

6. Solve :

(a) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)\,e^x \sec y$.

(b) $\left(D^2 - 2D + 1\right)y = xe^x \sin x$ 5+5

[*Turn over*]

7. (a) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, where λ is a parameter.

(b) Solve : $\sin y\frac{dy}{dx}=\cos x\left(2\cos y-\sin^2 x\right)$. 5+5