

Ex/CBCS/UG/GE/1.9/2018

BACHELOR OF ARTS EXAMINATION, 2018

(1st Year, 1st Semester)

MATHEMATICS - I

Full Marks : 30

Time : One Hour Thirty Minutes

The figures in the margin indicate full marks.

Use a separate Answer-Script for each part.

Symbols and Notations have their usual meaning.

Part - I

(15 Marks)

Answer any *three* questions.

5×3=15

1. If A, B, C be any three sub-sets of a set, then prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C). \quad 5$$

2. Suppose ρ be a relation in the set of integers $\mathbb{Z}$ defined by $a \rho b$ if and only if $(a - b)$ is divisible by 6. Then prove that ρ is an equivalence relation. 5

[Turn over]

[2]

3. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two mappings defined by $f(x) = x^2$ and $g(x) = x + 3$, for all $x \in R$,

Show that $f \circ g \neq g \circ f$. 5

4. State Euler's theorem for two variables.

If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, then

Prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. 5

Part - II

(15 Marks)

Answer any *three* questions. 5×3=15

5. Show that the function $f(x) = x^2 \sin \frac{1}{x}$, $x \neq 0$

$$f(0) = 0$$

is both continuous and derivable at $x = 0$. 5

6. State and prove Cauchy's mean value theorem. 5

[Turn over]

[3]

7. Discuss the applicability of the Mean Value theorem

$$f(b) - f(a) = (b - a)f'(\xi), \quad a < \xi < b, \text{ for}$$

$$f(x) = (4x - 5x^2 + x^3)/(1 - x); \quad -2 \leq x \leq 0.$$

Find ξ , if applicable. 5

8. Expand $f(x) = \cos x$ in a finite series in power of x , with remainder in Lagrange's form. 5

Or,

Find the values of a and b such that

$$\lim_{x \rightarrow 0} \frac{x(1 - a \cos x) + b \sin x}{x^3} = \frac{1}{3},$$

assuming that L' Hospital's rule is applicable. 5
