Ex/MSc/M/3.1/35/2018

MASTER OF SCIENCE EXAMINATION, 2018

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.1

(Advanced Topology)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols/Notations have their usual meanings.

Answer Question No. 1 and any *three* from the rest.

1. Define paracompactness and give an example of a topological space which is paracompact but not compact. 2

2. (a) Prove that a topological space (X, τ) is Hausdorff iff every convergent net in X has unique limit.

   (b) If $\mathscr{F}$ is a collection of subsets of X having finite intersection property then show that there is an ultrafilter $\hat{\mathscr{F}}$ containing $\mathscr{F}$.

[*Turn over*]

(c) Prove that following are equivalent

(i) (X, τ) is compact.

(ii) Every filter in X has a cluster point.

(iii) Every ultrafilter in X converges in X. 5+5+6

3. (a) Define a locally compact topological space. Prove that a locally compact Hausdorff space is completely regular. 1+6

(b) Prove that the evaluation map e of a family $\mathscr{F}$ of continuous functions on a topological space X is an open map provided $\mathscr{F}$ distinguishes points from closed sets. 4

(c) Let X be a topological space and for any two compactifications (f, Y) and (g, Z) of X define $(f, Y) \geq (g, Z)$ if there is a continuous mapping h from Y onto Z such that $h \circ f = g$. Prove that the collection of all T_2 compactifications of X is partially ordered by "$\geq$". 5

4. (a) Prove that every σ-locally finite cover of a topological space has a locally finite refinement covering X. 4

(b) Define a fully normal space and show that a fully normal space is normal. 1+3

(c) State and prove Arzela-Ascoli's Theorem. 8

5. (a) For a *metric space* prove that separability, 2nd countability and Lindeloffness are equivalent. 4

(b) Prove that $\mathbb{R}^w$ with the product topology is metrizable. Is the result true for an uncountable product of $\mathbb{R}$ endowed with product topology ? Justify your answer. 7+3

(c) Is the real line $\mathbb{R}$ endowed with lower limit topology metrizable ? Justify your answer. 2

6. (a) Define a uniform space. Give an example. 3

(b) Prove that a uniform space is pseudo-metrizable if its uniformity has a countable base. 5

(c) When a uniform space is said to be totally bounded ? Prove that a uniform space is compact iff it is totally bounded and complete. 1+7

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