

Ex/M.Sc./Math/22/4.5/B-2.19/2018

MASTER OF SCIENCE EXAMINATION, 2018

(2nd Year, 2nd Semester)

MATHEMATICS

Unit - 4.5 (B - 2.19)

(Graph Theory - II)

Full Marks : 30

Time : One Hour Thirty Minutes

All questions carry equal marks.

Notations / Symbols have their usual meanings.

Answer any *five* questions. $6 \times 5 = 30$

1. Define a plane graph G and its dual G^* . Let G be a plane graph. Show that G is bipartite if and only if G^* is Eulerian.
2. Define a minimal nonplanar graph. Prove that every minimal nonplanar graph is 2-connected.
3. Define the crossing number $\nu(G)$ of a graph G . Let G be a graph with n vertices and m edges. Let k be the maximum number of edges in a planar subgraph of G . Then show that

$$\nu(G) \geq \max \left\{ m - k, \frac{m^2}{2k} - \frac{m}{2} \right\}.$$

[Turn over]

[2]

4. Let G be a digraph with positive outdegree for each vertex. Then show that G contains a directed cycle.
 5. Let N be a transport network with a source s , a sink t and a flow F . Show that the flow out of s equals the flow into t .
 6. State and prove Hall's marriage theorem for matching in a bipartite graph.
 7. Find a maximum matching for the bipartite graph $G = (X, Y, E)$, where $X = \{u_1, u_2, u_3, u_4\}$,
 $Y = \{v_1, v_2, v_3, v_4\}$ and
 $E = \{u_1v_1, u_1v_2, u_1v_3, u_2v_1, u_3v_3, u_3v_4, u_4v_2, u_4v_4\}$.
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