Master of Science Examination, 2018

[1st Year, 2nd Semester]

MATHEMATICS

Unit - 2.2

(Topology)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Notations / Symbols have their usual meanings.

Answer Question No. 1 and any *three* from the rest.

1. Give an example of a mapping which is both open and closed but not continuous. 2

2. (a) In a topological space (X, τ) for any two sets $A, B \subset X$ prove that

(i) $\left(\overline{A \cup B}\right) = \overline{A} \cup \overline{B}$

(ii) $\overline{\overline{A}} = \overline{A}$ 3+4

[*Turn over*]

(b) Prove that a mapping $f:(X,\tau)\to(Y,\tau')$ where (X,τ) and (Y,τ') are topological spaces, is continuous iff f or any $A\subset X$, $f(\overline{A})\subset\overline{f(A)}$. 6

(c) Define the basis and a sub-basis of a topological space. Specify a basis and a subbasis of the real line with usual topology. 3

3. (a) When a topological space is called regular ? Prove that a topological space (X,τ) is regular iff for any $x\in X$ and open set $\bigcup$ containing x, there is an open set V such that $x\in V\subset\overline{V}\subset\bigcup$. 1+5

(b) Does regularity imply point separation i.e. T_0 or T_1 or T_2 ? Justify your answer. 1

(c) Prove that a 1st countable space is T_2 iff every convergent sequence has unique limit. 6

(d) When is a topological space called 2nd countable ? Give an example of a 1st countable topological space which is not 2nd countable. 3

4. (a) Prove that if E is connected and $E\subset T\subset\overline{E}$ then T is also connected. 3

(b) Prove that any subset of the real line with usual topology, containing at least two points is connected iff it is an interval. 7

(c) Define the path component C_p of a point p in a topological space. Prove that C_p is the largest pathwise connected set containing *p*. 6

5. (a) Prove that $(X \times Y,\ \tau \times \tau')$ is compact iff both (X, τ) and (Y, τ') are compact. 6

(b) Prove that a topological space (X, τ) is countably compact iff every infinite set in *X* has a ω-accumulation point. 6

(c) Prove that a countably compact first countable space is sequentially compact. 4

6. (a) If α and β are any two cardinal numbers then prove that either $\alpha \leq \beta$ or $\alpha \geq \beta$. 6

(b) For any set *A*, show that card $(\mathscr{P}(A))$ > Card (*A*). 5

(c) Give example of two sets with same cardinal number which have different order types. 2

(d) If *f* is an order isomorphism of a well ordered set onto itself then show that *f* must be the identity mapping. 3