

B.Sc. Mathematics 3rd Year 2nd Semester EXAMINATION 2024

MECHANICS

Full Marks: 40

Time: Two Hours

The figures in the margin indicate full marks.
(Symbols/Notations have their usual meanings)

Group - A

Any two questions

2X7=14

1. Two forces P and Q act along the straight lines whose equations are $y = x \tan \alpha, z = c$ and $y = -x \tan \alpha, z = -c$ respectively. Show that their central axis is $y = x \frac{P-Q}{P+Q} \tan \alpha, \frac{z}{c} = \frac{P^2-Q^2}{P^2+2PQ \cos 2\alpha + Q^2}$ for all values of P and Q also prove that this line is a generator of the surface

$$(x^2 + y^2)z \sin 2\alpha = 2cxy.$$

[CO2]

2. An isosceles triangular lamina is such that its mass per unit area at every point is proportional to the product of the distances of the point from equal sides of the triangle. Prove that the distance of the centre of gravity from the vertex is four-fifth of the altitude.

[CO3]

3. A body rests in equilibrium upon other fixed body, the portions of the two bodies in contact being spheres of radii r and R respectively and the straight line joining the centers of the spheres being vertical. If the first body be slightly displaced, find the conditions whether the equilibrium is stable or unstable, the body being rough enough to prevent sliding.

[CO2]

Group – B

Any two questions

2X7=14

4. A particle moves under the action of a central force F per unit mass in a medium whose resistance is equal to k times the velocity per unit mass. Prove that the differential equation of the path of the particle is

[Turn over

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h_0^2 u^2} e^{2kt},$$

where $u=1/r$ and h_0 is the initial moment of momentum of the particle per unit mass about the centre of force.

[CO3]

5 A particle falls down a smooth cycloid under its own weight starting from a cusp. Show that when it arrives to the vertex, the pressure of the curve on the particle is $2mg$, where m is the mass of the particle.

[CO4]

6. Describe the planetary motion under inverse square law. Discuss the nature of the conic by deriving the expression of velocity.

[CO1] +[CO3]

Group – C

Any two questions

2X6=12

7. Show that the moment of inertia of a right solid cone where height is 'h' and radius of whose base is 'a' is $\frac{3Ma^2}{20} \frac{6h^2+a^2}{h^2+a^2}$ about a slant side and $\frac{3M}{80}(h^2 + a^2)$ about a line through the centre of gravity of the cone and perpendicular to its axis.

[CO1]+[CO4]

8. Find the condition for which a given straight line will be a principal axis of a material system at a particular point. Also determine the other two principal axes. Further prove that, if the given straight line passes through the centre of gravity of the material then it will be principal axis at any point of its length.

[CO1]+[CO4]

9. A plank of mass M is initially at rest along a line of greatest slope of a smooth plank inclined at an angle ' α ' to the horizon and a man of mass M' , starting from the upper end walk down the plank so that so that it does not move. Show that he gets to the other end in

time $\sqrt{\frac{2M'a}{(M+M')g \sin \alpha}}$, where 'a' is the length of the plank.

[CO3]+[CO4]