B. Sc. Mathematics (Hons.) Examination, 2024

(3rd Year, 1st Semester)

Probability & Statistics

Paper : DSE-2A

Time : *Two Hours* | *Full Marks* : 40

(20 marks for each Group)

Symbols/Notations have their usual meanings.

Use separate Answer scripts for each Group.

GROUP—A

(20 Marks)

1. Answer ***any four*** questions : 5×4=20

(a) For two random variables X and Y, the random variables $U = X\cos\theta + Y\sin\theta$ and $V = Y\cos\theta - X\sin\theta$ are uncorrelated. Then prove that

$$\tan 2\theta = \frac{2\rho(X,Y)\,\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2},\ \sigma_x^2 \neq \sigma_y^2$$

(b) Let X be a continuous random variable having probability density function $f(x)$. Find the probability density function of $Y = 17X^2$.

(c) Two numbers are chosen independently at random from the interval (0,1). Find the probability that the sum of the number is greater than 1 but the sum of their square is less than 1.

(d) For Poisson distribution with parameter μ, find $E[x(x-1)(x-2)\cdots(x-n+1)]$, where X is a Poisson variate having parameter μ.

(e) State and prove Tchebycheff's inequality for a diserete random variable X having finite variance σ^2.

(f) Let $x_n \xrightarrow[in\,P]{} a$ and $y_n \xrightarrow[in\,P]{} b$ as $n \to \infty$.

Prove that

(i) $x_n - a \xrightarrow[in\,P]{} 0$ as $n \to \infty$

(ii) $cx_n \xrightarrow[in\,P]{} ca$ as $n \to \infty$

(iii) $x_n^2 \xrightarrow[in\,P]{} a^2$ as $n \to \infty$

One can use the result $x_n \pm y_n \xrightarrow[in\,P]{} a \pm b$ as $n \to \infty$.

GROUP—B

(20 Marks)

2. Answer ***any four*** questions : 5×4=20

(a) Define a sample and fake random variable. Show that probability distribution of a fake random variable is a satistical image of the distribution of the population.

(b) Show that sample variance is a consistent estimate of the population variance but it is not an unbiased estimate of the population variance.

(c) Let X is normally distributed with mean m and variance σ^2. Then the sampling distribution of sample mean $\overline{x}$ for a sample of size n is $N\left(m, \dfrac{\sigma}{\sqrt{n}}\right)$.

(d) Let $x_1, x_2, \ldots, x_{100}$ be a sample of size 100 drawn from a population having finite variance σ^2. Then determine the constant d such that

$$d[(x_1 - x_2)^2 + (x_3 - x_4)^2 + \cdots + (x_{99} - x_{100})^2]$$

is an unbiased estimate of σ^2.

(e) If the value of m is known, find the maximum likelihood estimate of σ^2 for a normal population and show that the estimate is unbiased and consistent.

(f) (i) Show that

Power of the test = 1 – p (Type-II error)

(ii) Let $f(x)$ be the probability density function of the population of a random variable X, where

$$f(x) = \begin{cases} \dfrac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{elsewhere} \end{cases}$$

to test the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$, a sample of size 1 is drawn from the population of X. If $w = \{x : 0{\cdot}5 \le x\}$ is the critical region, find the power of the test.

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